

# Results of the 2013 IEEE CEC Competition on Niching Methods for Multimodal Optimization

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# Outline

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# Introduction

- Numerical optimization is probably one of the most important disciplines in optimization
- Many real-world problems are “**multimodal**” by nature, i.e., multiple satisfactory solutions exist
- **Niching methods:** promote and maintain formation of multiple stable subpopulations within a single population
  - **Aim:** maintain diversity and locate multiple globally optimal solutions.
- **Challenge:** Find an efficient optimization algorithm, which is able to **locate multiple global optimal solutions** for multimodal problems with various characteristics.

# Competition

Provide a common platform that encourages fair and easy comparisons across different niching algorithms.

X. Li, A. Engelbrecht, and M.G. Epitropakis, “Benchmark Functions for CEC’2013 Special Session and Competition on Niching Methods for Multimodal Function Optimization”, Technical Report, Evolutionary Computation and Machine Learning Group, RMIT University, Australia, 2013

- 20 benchmark multimodal functions with different characteristics
- 5 accuracy levels:  $\varepsilon \in \{10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\}$
- The benchmark suite and the performance measures have been implemented in: C/C++, Java, MATLAB

# Benchmark function set

X. Li, A. Engelbrecht, and M.G. Epitropakis, “[Benchmark Functions for CEC'2013 Special Session and Competition on Niching Methods for Multimodal Function Optimization](#)”, Technical Report, Evolutionary Computation and Machine Learning Group, RMIT University, Australia, 2013

Id	Dim.	# GO	Name	Characteristics
$F_1$	1	2	Five-Uneven-Peak Trap	Simple, deceptive
$F_2$	1	5	Equal Maxima	Simple
$F_3$	1	1	Uneven Decreasing Maxima	Simple
$F_4$	2	4	Himmelblau	Simple, non-scalable, non-symmetric
$F_5$	2	2	Six-Hump Camel Back	Simple, not-scalable, non-symmetric
$F_6$	2,3	18,81	Shubert	Scalable, #optima increase with D, unevenly distributed grouped optima
$F_7$	2,3	36,216	Vincent	Scalable, #optima increase with D, unevenly distributed optima
$F_8$	2	12	Modified Rastrigin	Scalable, #optima independent from D, symmetric
$F_9$	2	6	Composition Function 1	Scalable, separable, non-symmetric
$F_{10}$	2	8	Composition Function 2	Scalable, separable, non-symmetric
$F_{11}$	2,3,5,10	6	Composition Function 3	Scalable, non-separable, non-symmetric
$F_{12}$	2,3,5,10	8	Composition Function 4	Scalable, non-separable, non-symmetric

## Measures:

**Peak Ratio** (PR) measures the average percentage of all known global optima found over multiple runs:

$$\text{PR} = \frac{\sum_{run=1}^{NR} \# \text{ of Global Optima}_i}{(\# \text{ of known Global Optima}) * (\# \text{ of runs})}$$

## Who is the winner:

- The participant with the highest average Peak Ratio performance on all benchmarks **wins**.
- In all functions the following holds: the higher the PR value, the better

# Participants

## Submissions to the competition:

- E-1682: (**PNA-NSGAII**) A Parameterless-Niching-Assisted Bi-objective Approach to Multimodal Optimization
- E-1419: (**N-VMO**) Variable Mesh Optimization for the 2013 CEC Special Session Niching Methods for Multimodal Optimization
- E-1449: (**dADE/nrand/1,2**) A Dynamic Archive Niching Differential Evolution algorithm for Multimodal Optimization
- Mike Preuss: (**NEA1, NEA2**) Niching the CMA-ES via Nearest-Better Clustering [2]

# Participants (2)

Implemented algorithms for comparisons:

- **(A-NSGAII)** A Bi-objective NSGA-II for multimodal optimization (taken from E-1682)[1]
- **(CrowdingDE)** Crowding Differential Evolution [3]
- **(DECG, DELG, DELS-aj)** [4]
- **(DE/nrand/1,2)** Niching Differential Evolution algorithms with neighborhood mutation strategies [5]
- **(CMA-ES, IPOP-CMA-ES)** CMA-ES/IPOP-CMA-ES with a restart procedure and a dummy archive. [6,7]

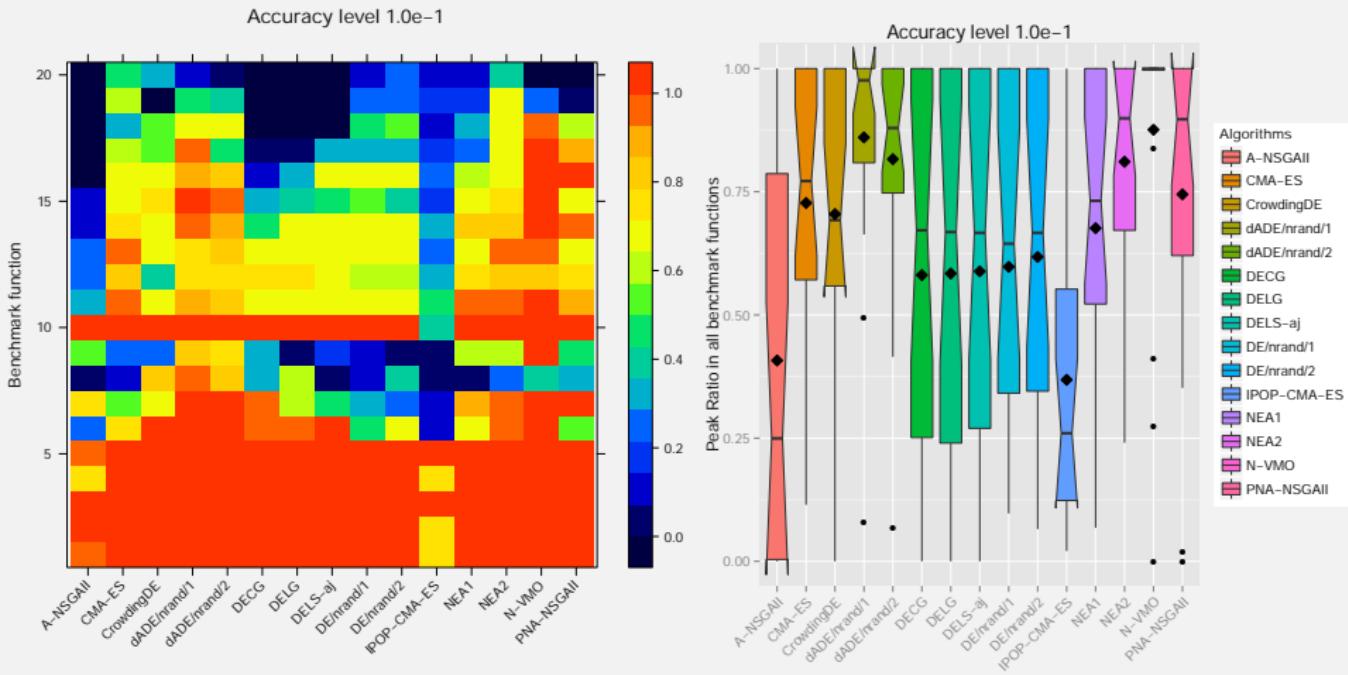
Mike Preuss: CMA-ES, IPOP-CMA-ES, MG Epitropakis:  
DE/nrand/1,2, DECG, DELG, DELS-aj, CrowdingDE

# Results

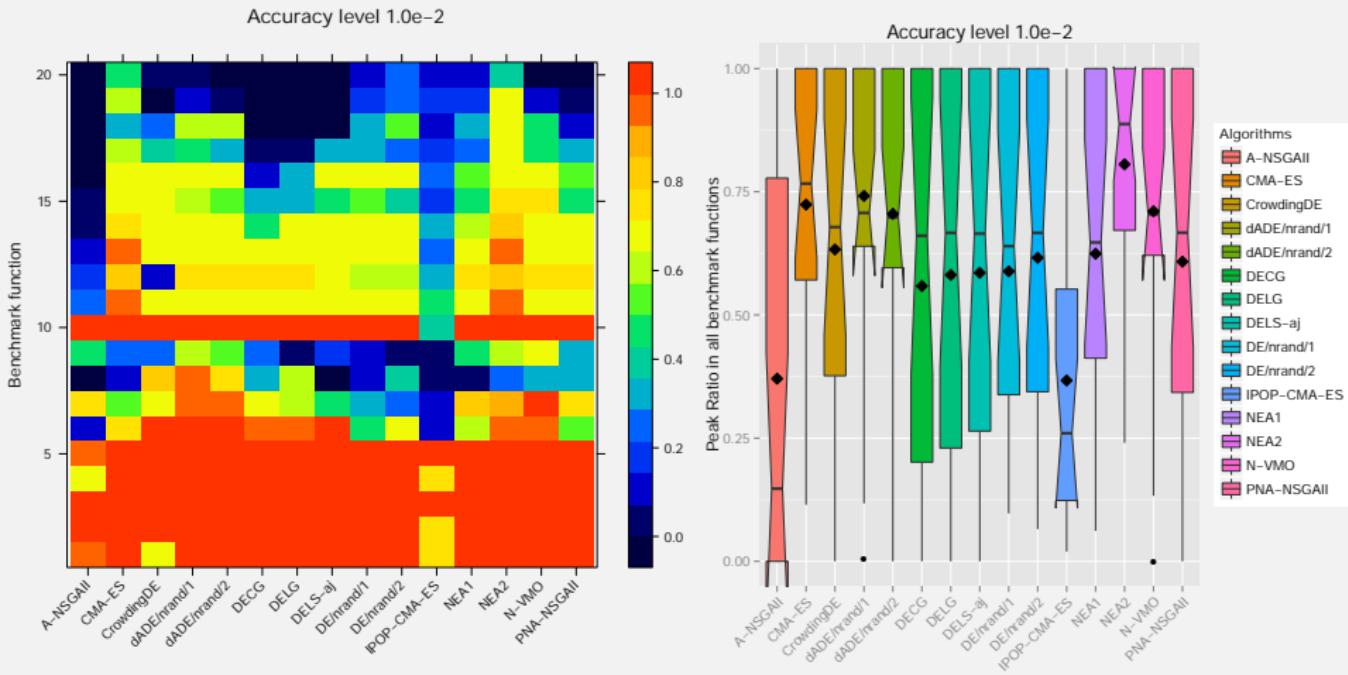
## Summary:

- 4 submissions/teams from six countries (four continents)
- 15 algorithms
- 20 benchmark functions
- 5 accuracy levels  $\varepsilon \in \{10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\}$
- Results: **per accuracy level & over all accuracy levels**

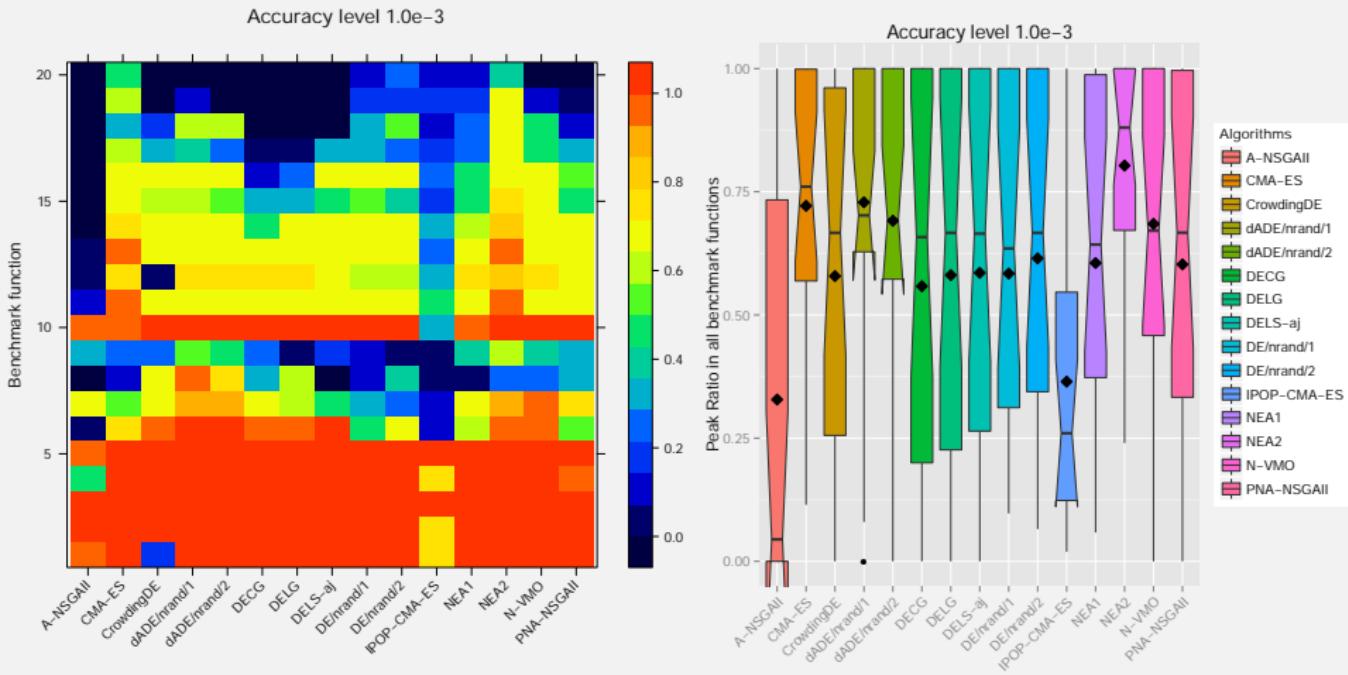
# Accuracy level $\varepsilon = 10^{-1}$



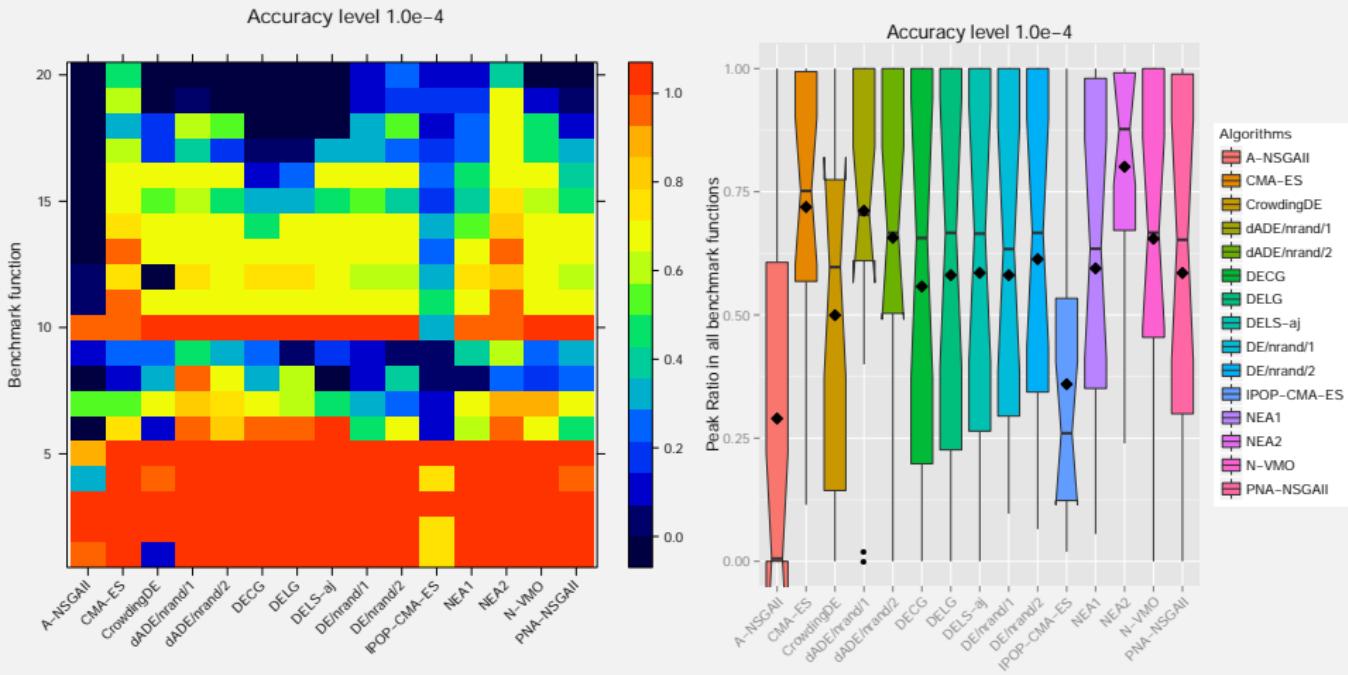
# Accuracy level $\varepsilon = 10^{-2}$



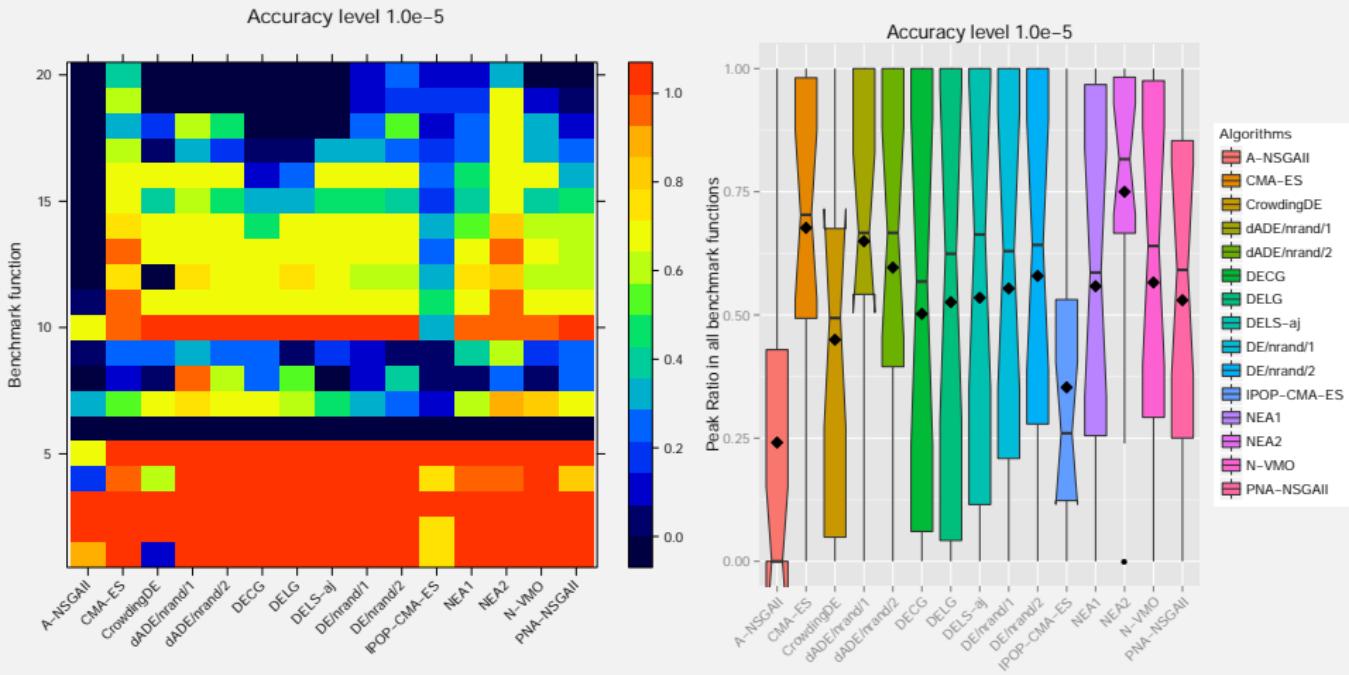
# Accuracy level $\varepsilon = 10^{-3}$



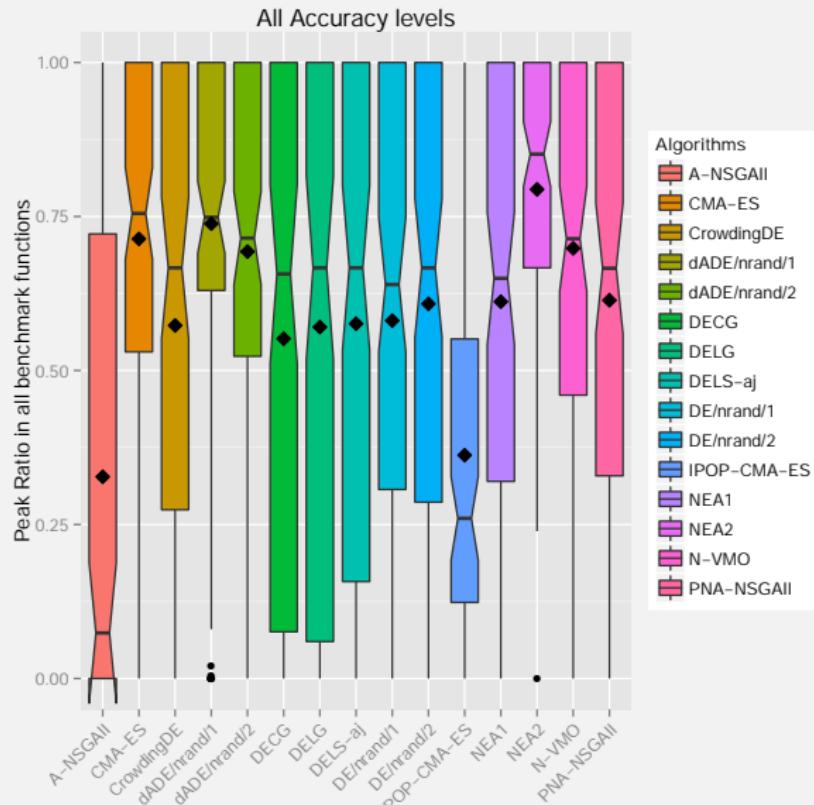
# Accuracy level $\varepsilon = 10^{-4}$



# Accuracy level $\varepsilon = 10^{-5}$



# Overall performance (1)



# Overall performance (2)

Algorithm	Statistics			Friedman's Test	
	Median	Mean	St.D.	Rank	Score
A-NSGAI	0.0740	0.3275	0.4044	15	3.1450
CMA-ES	<b>0.7550</b>	<b>0.7137</b>	0.2807	<b>3</b>	<b>10.2300</b>
CrowdingDE	0.6667	0.5731	0.3612	8	7.7900
dADE/nrand/1	<b>0.7488</b>	<b>0.7383</b>	0.3010	<b>2</b>	<b>10.6700</b>
dADE/nrand/2	0.7150	0.6931	0.3174	5	9.6200
DECG	0.6567	0.5516	0.3992	13	6.4950
DELG	0.6667	0.5706	0.3925	11	7.0350
DELS-aj	0.6667	0.5760	0.3857	12	7.0250
DE/nrand/1	0.6396	0.5809	0.3338	9	7.7600
DE/nrand/2	0.6667	0.6082	0.3130	6	8.3200
IPOP-CMA-ES	0.2600	0.3625	0.3117	14	3.8900
NEA1	0.6496	0.6117	0.3280	10	7.6300
NEA2	<b>0.8513</b>	<b>0.7940</b>	0.2332	<b>1</b>	<b>11.9300</b>
N-VMO	<b>0.7140</b>	<b>0.6983</b>	0.3307	<b>4</b>	<b>10.1550</b>
PNA-NSGAI	0.6660	0.6141	0.3421	7	8.3050

# Winners

## Ranking based on average PR values

- ① **NEA2** (Mike Preuss) Niching the CMA-ES via Nearest-Better Clustering
- ② **dADE/nrand/1** (E-1449) A Dynamic Archive Niching Differential Evolution algorithm
- ③ **CMA-ES** (Mike Preuss) CMA-ES with simple archive
- ④ **N-VMO** (E-1419) Niching Variable Mesh Optimization algorithm

Note: The algorithms have not been fine-tuned for the specific benchmark suite!

# Conclusions

## Summary

- Four teams from six countries (four continents)
- **Winner: NEA2** (Mike Preuss) Niching the CMA-ES via Nearest-Better Clustering
  - Competitive on average performance, (nearest-better clustering, archive mechanism, CMA-ES)
- Places 2 to 4 very close:
  - **dADE/nrand/1** (E-1449) A Dynamic Archive Niching Differential Evolution algorithm
  - **CMA-ES** (Mike Preuss) CMA-ES with simple archive
  - **N-VMO** (E-1419) Niching Variable Mesh Optimization algorithm

# Conclusions (2)

- The competition gave a boost to the multimodal optimization community
- New competitive and very promising approaches

## Key characteristics of the algorithms:

- Many attempts to overcome the influence of the algorithm's parameters (niching parameters, population size)
- Usage of Archives to maintain good solutions
- Multiobjectivization, Clearing, Clustering and neighborhood mutation-based niching techniques
- Algorithms: Differential Evolution, CMA-ES, Variable Mesh Optimization and NSGAII

# Future Work

## Possible objectives:

- Re-organize the competitions in future
- Enhance the benchmark function set
- Introduce new performance measures
- Automate the experimental design and results output
- Boost multimodal optimization community

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- Dr. Catalin Stoean, University of Craiova, Romania

(-: Thank you very much for your attention :-)



Questions ???

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