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# Symbol detection in spatial multiplexing system using particle swarm optimization meta-heuristics

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SUMMARY

Symbol detection in multi-input multi-output (MIMO) communication systems using different particle swarm optimization (PSO) algorithms is presented. This approach is particularly attractive as particle swarm intelligence is well suited for real-time applications, where low complexity and fast convergence is of absolute importance. While an optimal maximum likelihood (ML) detection using an exhaustive search method is prohibitively complex, PSO-assisted MIMO detection algorithms give near-optimal bit error rate (BER) performance with a significant reduction in ML complexity. The simulation results show that the proposed detectors give an acceptable BER performance and computational complexity trade-off in comparison with ML detection. These detection techniques show promising results for MIMO systems using high-order modulation schemes and more transmitting antennas where conventional ML detector becomes computationally non-practical to use. Hence, the proposed detectors are best suited for high-speed multi-antenna wireless communication systems. Copyright © 2008 John Wiley & Sons, Ltd.

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## 1. INTRODUCTION

The demand for higher data rate communications for multimedia-based bandwidth-intensive applications is on increase. Multi-input-multi-output (MIMO)-based systems have shown promise to meet these challenges [1, 2]. Efficient exploitation of spatial diversity available in the MIMO channel enables higher system capacity. Orthogonal frequency division multiplexing (OFDM)

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employed in conjunction with MIMO architecture constitutes an attractive solution for modern wireless communication systems [3] as it has the ability to deal with multipath propagation. It effectively takes advantage of random fading [1–4] and multipath delay spread [5, 6]. A number of architectures have been developed for MIMO symbol detection. The Vertical Bell Laboratories LAyered Space–Time (VBLAST) system is one successful implementation of MIMO systems [1, 2]. However, the performance of the VBLAST detection scheme [7, 8] is limited due to imperfect interference cancellation and insufficient receive diversity. The ML detection scheme performs the best, but its complexity increases exponentially with transmit antennas and with higher modulation schemes [9–18].

One of the challenges in building wide-band MIMO systems is the tremendous processing power required at the receiver side. Although coded MIMO schemes offer better performance than separate channel coding and modulation scheme by fully exploring the trade-off between multiplexing and diversity [19], its hardware complexity can be significant, especially for wide-band system with more than four antennas at both the transmitter and the receiver sides. On the other hand, it is easier to implement traditional channel-coding schemes such as Convolution code and Turbo code for data rates of hundreds of Mbps. For this reason, we discuss the uncoded MIMO system also called spatial multiplexing.

Several MIMO detection techniques have been proposed [9–21] to reduce its complexity. Group detection schemes [20] give better performance but their complexity increases, when the number of sub-streams in a group is large. Soft interference cancellation mitigates error propagation effect; however, complexity is still a problem [21]. Another potential solution is sphere decoder (SD) [10–14], which has polynomial computational cost on the average [11]. However, when problem dimensions are high, its complexity coefficients and variance of computational time become large. In [17] QR decomposition with sort and Dijkstra's algorithm is used to decrease the complexity of SD. The multistage likelihood scheme is proposed in [18] where the Euclidean distance of the candidate symbol combinations is calculated, instead of all possible combinations.

Real-life optimization problems (such as ML detection) are often so complex that finding the best solution becomes computationally infeasible. Therefore, an intelligent approach is to search for a good approximate solution with reduced complexity. Many techniques have been proposed that imitate nature's own ingenious ways to explore the optimal solution. The earliest of the nature-inspired techniques are the genetic and other evolutionary heuristic algorithms that evoke Darwinian evolution principles.

Swarm intelligence (SI) [22–25] is one such innovative distributed intelligent paradigm for solving optimization problems, which originally took its inspiration from the biological examples such as swarming and flocking phenomena in vertebrates.

Particle swarm optimization (PSO) meta-heuristics is a population-based SI technique inspired by the coordinated movements of birds flocking introduced by Kennedy and Eberhart in [26]. Standard PSO (SPSO) uses a real-valued multidimensional solution space, whereas in binary PSO (BPSO) particle positions are binary rather than real valued [27]. The combination of the pure heuristics such as PSO with local search (LS) is termed as 'memetic algorithms' (MAs) [28]. MAs are extensions that apply additional procedure to further refine the search results efficiently. This hybridization improves search efficiency [29]. An application of PSO for symbol detection in the MIMO system has been proposed in [30, 31].

In this paper, reduced complexity near-ML detection using PSO techniques is presented. The remainder of this paper is organized as follows. Section 2 formulates the MIMO symbol



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detection problem for flat-fading channel. A brief account on existing MIMO detection techniques is presented in Section 3. Section 4 explains the proposed PSO-MIMO detection algorithms, followed by performance analysis in Section 5. The paper is concluded in Section 6.

## 2. MIMO DETECTION FOR FLAT-FADING CHANNEL

### *2.1. MIMO channel model*

Consider a MIMO system as shown in Figure 1, where $N_t$ different signals are transmitted and arrive at an array of $N_r (N_t \leqslant N_r)$ receivers via a rich-scattering flat-fading environment. Grouping all the transmitted and received signals into vectors, the system can be viewed as transmitting an $N_t \times 1$ vector signal $\mathbf{x}$ through an $N_t \times N_r$ matrix channel $\mathbf{H}$, with $N_r \times 1$ Gaussian noise vector $\mathbf{v}$ added at the input of the receiver. The received signal as an $N_r \times 1$ vector can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v} \tag{1}$$

where $\mathbf{y}$ is the received $N_r \times 1$ vector. The $(n_r, n_t)$th element of $\mathbf{H}$ $h_{n_r n_t}$ is the complex channel response from the $n_t$th transmit antenna to the $n_r$th receive antenna. The transmitted symbol $\mathbf{x}$ is zero mean and has covariance matrix $\mathbf{R_x} = E\{\mathbf{x}\mathbf{x}^*\} = \sigma_x^2 \mathbf{I}$. The vector $\mathbf{v}$ is also zero mean and $\mathbf{R_v} = E\{\mathbf{v}\mathbf{v}^*\} = \sigma_v^2 \mathbf{I}$. In frequency-selective fading channels, the entire channel frequency response $h_{n_r n_t}$ is no longer characterized by a constant; therefore, we can write it as a function of the frequency

$$\mathbf{y}(f) = \mathbf{H}(f)\mathbf{x}(f) + \mathbf{v}(f) \tag{2}$$

When OFDM modulation is used, the entire channel is divided into a number of sub-channels. These sub-channels are spaced orthogonally to each other such that no inter-carrier interference is present at the sub-carrier frequency subject to perfect sampling and carrier synchronization. When sampled at the sub-carrier frequency of $f_{n_c}$, the channel model becomes

$$\mathbf{y}^{(n_c)} = \mathbf{H}^{(n_c)}\mathbf{x}^{(n_c)} + \mathbf{v}^{(n_c)}, \quad n_c = -N_c/2, \ldots, N_c/2 - 1 \tag{3}$$

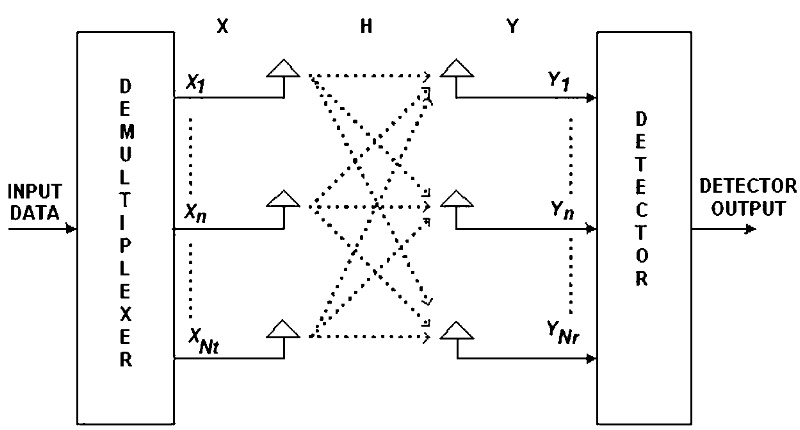


Figure 1. $N_t \times N_r$ MIMO communications system model.



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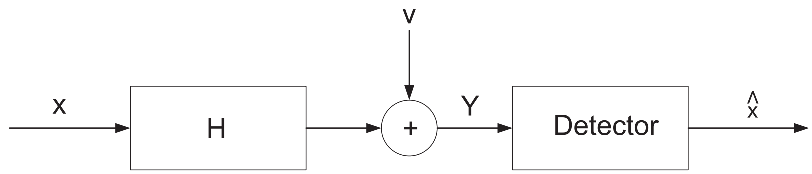


Figure 2. A simplified linear MIMO communication system showing the following discrete signals: transmitted symbol vector $\mathbf{x}\in\chi^{N_t}$, channel matrix $\mathbf{H}\in\mathbb{R}^{N_t\times N_r}$, additive noise vector $\mathbf{v}\in\mathbb{R}^{N_t}$, receive vector $\mathbf{y}\in\mathbb{R}^{N_t}$ and detected symbol vector $\hat{\mathbf{x}}\in\mathbb{R}^{N_r}$.

With $N_c$ sufficiently large, the sub-channel at each of the sub-carriers can be regarded as flat fading. Therefore, when using OFDM, the MIMO detection over frequency-selective channels is transformed into MIMO detection over $N_c$ narrow band flat-fading channels. For this reason, we only focus on the MIMO detection algorithms in flat-fading channels. The entries of the channel matrix $\mathbf{H}$ are assumed to be known to the receiver but not to the transmitter. This assumption is reasonable if training or pilot signals are sent to estimate the channel, which is constant for some coherent interval.

### 2.2. *Problem formulation*

The task is that of detecting $N_t$ transmitted symbols from a set of $N_r$ observed symbols that have passed through a non-ideal communication channel, typically modeled as a linear system followed by an AWGN as shown in Figure 2.

Transmitted symbols from a known finite alphabet $\boldsymbol{\chi}=\{x_1,\ldots,x_M\}$ of size $M$ are passed to the channel. The detector chooses one of the $M^{N_t}$ possible transmitted symbol vectors from the available data. Assuming that the symbol vectors $\mathbf{x}\in\chi^{N_t}$ are equiprobable, the *maximum likelihood* (*ML*) detector always returns an optimal solution according to the following:

$$\mathbf{x}_* \triangleq \arg\max_{x\in\chi^{N_t}} P(\mathbf{y}\text{ is observed}|\mathbf{x}\text{ was sent}) \tag{4}$$

Assuming the additive noise $\mathbf{v}$ to be white and Gaussian, the ML detection problem of Figure 2 can be expressed as the minimization of the squared Euclidean distance to a target vector $\mathbf{y}$ over $N_t$-dimensional finite discrete search set

$$\mathbf{x}_* = \arg\min_{x\in\chi^{N_t}} \|\mathbf{y}-\mathbf{H}\mathbf{x}\|^2 \tag{5}$$

The optimal ML detection scheme needs to examine all $M^{N_t}$ or $2^{bN_t}$ symbol combinations ($b$ is the number of bits per symbol). The problem can be solved by enumerating over all possible $\mathbf{x}$ and finding the one that causes the minimum value as in (5). Therefore, the computational complexity increases exponentially with constellation size $M$ and the number of transmitters $N_t$.

We present PSO algorithms-assisted MIMO-OFDM symbol detectors thus viewing the MIMO symbol detection issue as a combinatorial optimization and approximate the near optimal solution iteratively with lesser than ML computational complexity. In the following section, a brief note on some of the existing MIMO detection techniques is presented.



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## 3. EXISTING MIMO DETECTORS

### *3.1. Linear MIMO detectors*

A straightforward approach to recover $x$ from $y$ is to use an $N_t \times N_r$ weight matrix $\mathbf{W}$ to linearly combine the elements of $\mathbf{y}$ to estimate $\mathbf{x}$, i.e. $\hat{x} = \mathbf{W}\mathbf{y}$. Zero-forcing (ZF) and minimum mean-squared error (MMSE) are linear detectors. The ZF algorithm attempts to null out the interference introduced from the matrix channel by directly inverting the channel with the weight matrix [9]. A drawback of ZF is that nulling out the interference without considering the noise can increase the noise power significantly, which in turn results in performance degradation. To solve this, MMSE minimizes the mean-squared error, i.e. $J(\mathbf{W}) = E\{(x-\hat{x})*(x-\hat{x})\}$, with respect to $\mathbf{W}$ [32, 33]. MMSE possess the desirable property of not enhancing noise as much as ZF. Furthermore, its bit error rate (BER) performance is better than ZF.

### *3.2. Nonlinear MIMO detectors*

A popular nonlinear combining approach is the VBLAST [1, 2] known as ordered successive interference cancellation. It uses the detect-and-cancel strategy similar to that of decision-feedback equalizer. Either ZF or MMSE can be used for detecting the strongest signal component used for interference cancellation. The performance of this procedure is generally better than those of ZF and MMSE. VBLAST still provides a suboptimal solution with lower computational complexity than ML. However, the performance of VBLAST is degraded due to error propagation.

### *3.3. ML detectors*

ML detector is optimal but computational complexity as given in (5) is extremely high [18]; therefore, it is a not practical approach in MIMO systems using many transmitters and higher QAM constellations.

## 4. PARTICLE SWARM OPTIMIZATION FOR MIMO-OFDM SYSTEM

### *4.1. Particle swarm optimization*

PSO argues that intelligent cognition derived from interactions of individuals in a social world and this socio-cognitive approach can be effectively applied to computationally intelligent systems [23]. A swarm consists of a number of particles (possible solutions) that move (fly) through the feasible solution space to explore the optimal solution that can be coded as binary strings or real-valued vectors. The particles are capable of interacting with each other in a given neighborhood and traverse a search space where a quality measure, fitness, can be evaluated. The particles are evolved through cooperation and competition among themselves over iterations. The coordinates of each particle represent a possible solution associated with two vectors, position ($\mathbf{X}_i$) and velocity ($\mathbf{V}_i$). In $d$-dimensional search space, the $i$th particle can be represented by $d$-dimensional position vector $\mathbf{X}_i = (x_{i1}, x_{i2}, \ldots, x_{id})$ and another $d$-dimensional velocity vector $\mathbf{V}_i = (v_{i1}, v_{i2}, \ldots, v_{id})$. Each particle experiences an iterative procedure of adaptation to two types of major information, i.e. individual learning and cultural transmission, which means the procedure accelerates particles at each time step towards personal best (best value recorded by each particle) and the position of the most recent global best point (best position returned form the swarm), with the relative



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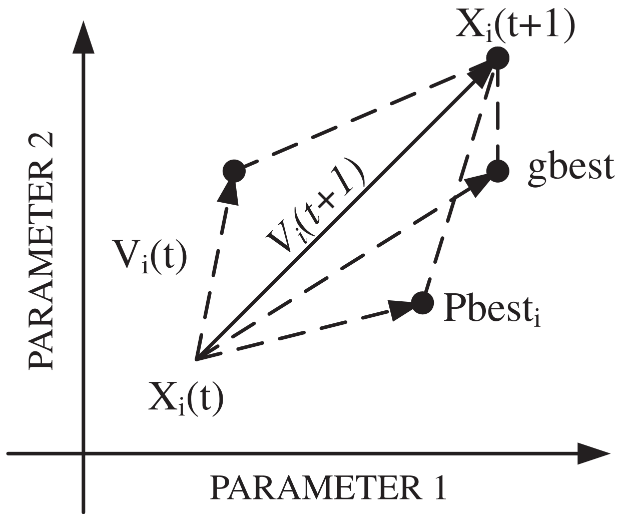


Figure 3. Vector representation of PSO model.

acceleration towards each determined stochastically. A key attractive feature of the PSO approach is its simple mathematical model involving two model equations [23] and fewer parameters to adjust. In BPSO [27] velocity loses its physical meaning. It is used to determine a probability by squashing velocities to the range $(0, 1)$ by using a sigmoid function. A proof of explicit PSO equations and its guaranteed convergence [25] suggests its use as ML function optimizer as discussed below.

### *4.2. PSO-MIMO detection algorithm*

We exploit parsimonious PSO algorithm's potential to optimize symbol detection in the MIMO system. An important step to implement PSO is to define a *fitness function*; this is the link between the optimization algorithm and the real-world problem. The fitness function is unique for each optimization problem. The fitness function using the coordinates of the particle returns a fitness value to be assigned to the current location. If the value is greater than the value at respective personal best (pbest) for each particle, or global best (gbest) for the swarm, then previous locations are updated with the present locations. The velocity of the particle is changed according to the relative locations of pbest and gbest as shown in Figure 3. Once the velocity of the particle is determined, it simply moves to the next position. After this process is applied on each particle, it is repeated till the maximum number of iterations is reached. The PSO algorithm's flow diagram is shown in Figure 4. This exploratory-exploitive optimization approach can be extended to the MIMO detection optimization problem.

The major challenge in designing PSO-based MIMO detectors is selection of algorithm parameters that fit the symbol detection optimization problem. Selection of effective fitness function is also vital and problem dependent. The fitness function perhaps is the only link between the real-world problem and the optimization algorithm. The basic fitness function used by the optimization algorithm to converge to the near optimal solution is (5), which is minimum Euclidian distance. In addition, the choice of initial solution guess plays an important role in the fast convergence to a suitable solution. Initial guess is essential for these algorithms to perform well. Therefore,



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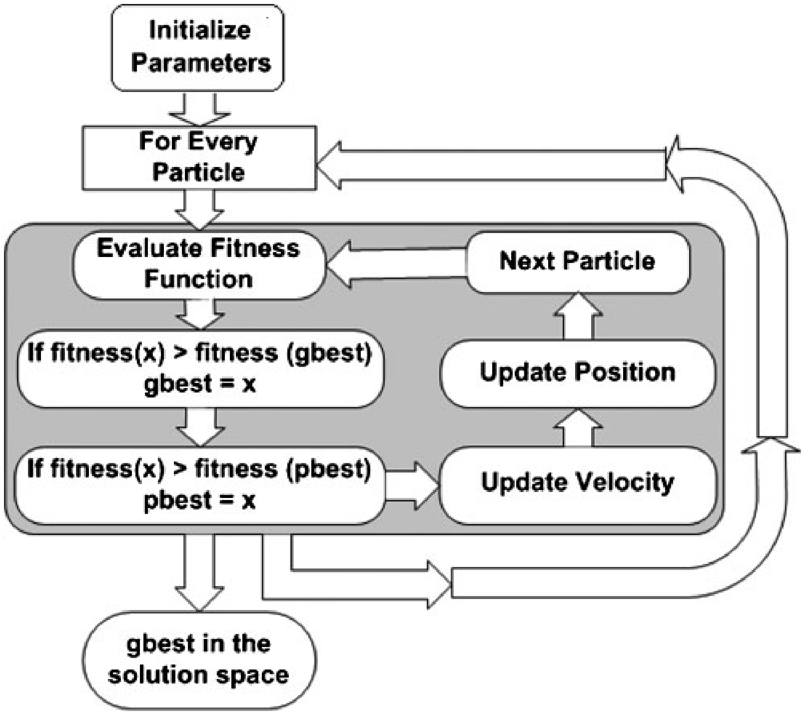


Figure 4. PSO algorithm's flow diagram.

the proposed PSO-MIMO detector takes the output of ZF or ZF-VBLAST as its initial solution guess. This educated guess enables the algorithm to reach a more refined solution iteratively by ensuring fast convergence. Assuming random initialization does not guarantee convergence in few iterations.

### 4.3. *SPSO-MIMO detection algorithm*

The proposed MIMO detection algorithm based on standard continuous PSO [30] is described below:

(1) Initialize the particle size (swarm) by taking the initial solution guess. Initialize the algorithm parameters.
(2) Fitness of each particle with is a potential candidate solution is calculated using (5):

$$f = \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 \tag{6}$$

Minimum Euclidean distance for each symbol represents the fitness of solution. Find the global best performance '$\text{gbest}_d$' in the population that represents the least Euclidean distance found so far. Record the personal best '$\text{pbest}_{id}$' for each bit along its previous values.



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(3) Velocity for each particle is computed using the following PSO velocity update equation:

$$\mathbf{v}_{id}(k)=\mathbf{v}_{id}(k-1)+\varphi_1\text{rand}_1[\text{pbest}_{id}-x_{id}(k-1)]+\varphi_2\text{rand}_2[\text{gbest}_d-x_{id}(k-1)] \tag{7}$$

with $\mathbf{v}_{id}\in\{-v_{\max}, v_{\max}\}$.

(4) The particle position is updated depending on the following PSO velocity update equation:

$$x_{id}(k)=x_{id}(k-1)+v_{id}(k) \tag{8}$$

(5) Repeat from step 2 until maximum number of iterations is reached. Here '$k$' is the number of iterations. An optimum number of iterations are tuned for efficient performance. The solution gets refined iteratively.

### *4.4. MPSO-MIMO detection algorithm*

Hybridization of SPSO with LS is termed as 'memetic' PSO (MPSO). The MPSO procedure further refines the solution found out by SPSO-MIMO using lower significant bit (LSB) flipping. The algorithm is explained below:

(1) Initialize the algorithm parameters. Assume the initial solution guess.
(2) Find fitness using (6). Find '$\text{gbest}_d$' and '$\text{pbest}_{id}$'. Perform velocity and position for each particle using (7) and (8).
(3) Apply neighborhood search, by initializing '$b_s$'. Evaluate the fitness of neighbors iteratively; update '$\text{gbest}_d$' and '$\text{pbest}_{id}$'.
(4) Repeat from step 2 until the maximum number of iterations is reached.

Pseudo-code is shown in Table I. The degree of LS '$b_s$' indicates the LSBs of the best solution found so far. Keeping the search degree as two would mean four neighbors would be searched for better fitness. In our LS algorithm '$b_s$' is kept as 1 and 2, respectively.

### *4.5. BPSO-MIMO detection algorithm*

The application of binary version of PSO for symbol detection in the MIMO system results in a further improved performance [31]. Here the particles are binary rather than real valued as in the earlier case of the SPSO-MIMO algorithm. The proposed BPSO-based MIMO detection algorithm is explained below:

(1) Take the output of ZF or ZF-VBLAST such as $\mathbf{x}_i\in\{0,1\}$ as initial particles (initial solution bit string) instead of selecting randomly from the solution space.
(2) The algorithm parameters are initialized. '$v_{id}$' is initialized to zero (equal probability for binary decision), '$\text{pbest}_{id}$' and '$\text{gbest}_d$' are initialized to maximum Euclidean distance depending upon the QAM size.
(3) Evaluate the fitness of each particle (bit) using the same fitness function of (6). The effect on the Euclidean distance due to search space bits is measured. Find the global best performance '$\text{gbest}_d$' in the population and record the personal best '$\text{pbest}_{id}$' for each bit along its previous values.
(4) For each search space bit at the $d$th side of the bit string of particle $\mathbf{x}_i$, compute the bits velocity using the PSO velocity update Equation (7).



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Table I. MPSO-MIMO algorithm's pseudo-code.

```
Phase-1: Initialize particle size = N_p; Randomly get particle
position; Set boundaries for velocities; Initialize parameters;

Phase-2: For i = 1:N_p
    %Get velocity for next position updating
v_id(k) = v_id(k-1) + φ1 rand1[pbest_id - x_id(k-1)] + φ2 rand2[gbest_d - x_id(k-1)]
        If velocity < boundary
        %Update particle position;
                x_id(k) = x_id(k-1) + v_id(k)
        Else
        %Velocity = Cyclic velocity;
        End
        %Apply neighborhood search
        Initialize lower bits to search
        For J = 1:Number of neighbors
            %Iterative search in neighborhood
            Evaluate();
            If Fitness(x_id(k)) < Fitness(x_id(k-1))
            %Update Local Best;
             p_besti = x_id(k);
        End
            %Update Global Best;
            G_best = min(p_best);
        End
```

(5) The particle position is updated depending on the following binary decision rule:

$$\text{If } \mathrm{rand}_3 < \mathbf{S}(\mathbf{v}_{id}(\mathbf{k})) \quad \text{then } \mathbf{x}_{id}(\mathbf{k}) = 1, \text{ else } \mathbf{x}_{id}(\mathbf{k}) = 0 \tag{9}$$

(6) Goto step 3 until maximum number of iterations is reached.

'rand' is a random number generated uniformly in [0,1] and '$S$' is the sigmoid transformation function

$$S(v_{id}(k)) = \frac{1}{1+\exp(-v_{id}(k))} \tag{10}$$

The parameter '$v_i$' is the particles predisposition to make 1 or 0; it determines the probability threshold to make this choice. The individual is more likely to choose 1 for higher $v_{id}(k)$, whereas its lower values will result in the choice of 0. Such a threshold needs to stay in the range of [0, 1]. The sigmoid logistic transformation function maps the value of $v_{id}(k)$ to a range of [0, 1].

### *4.6. PSO parameter control*

The terms $\phi_1$ and $\phi_2$ are positive acceleration constants used to scale the contribution of cognitive and social components such that $\phi_1 + \phi_2 < 4$ [9]. These are used to stochastically vary the relative pull of pbest and gbest. $v_{\max}$ sets a limit to further exploration after the particles have converged. Its values are problem dependent but usually set in the range of $\pm 4$ for BPSO and $\pm 10$ for standard PSO [9]. The particle size is assumed fixed for SPSO and MPSO; however, it varies with the system in the case of BPSO. These parameters are discussed next.



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### 4.7. PSO-MIMO detection algorithm's relationship

In SPSO originally proposed in [26] particle positions are real valued; however, in its later version BPSO [27], the particles represent binary bits rather than real values. Therefore, the MIMO detection problem formulation using SPSO as discussed in Section 4.3 is based on real-valued search space, whereas, in the case of the BPSO-MIMO detection algorithm as explained in Section 4.5, the problem formulation is different. Here the solution is converted into binary string and particles are bits in this string. The BPSO algorithm is applied on each bit to get the newer solution bit string for fitness analysis.

The MPSO-based MIMO detection algorithm finds out the solution using exactly the same procedure as that of the SPSO-MIMO algorithm; however, the solution returned is further refined using the LS method. Therefore, the MPSO-MIMO detection technique offers better performance with additional complexity overhead in comparison with the SPSO-MIMO detection algorithm. In depth performance analysis of these proposed PSO-MIMO detection methods is presented below.

## 5. SIMULATION RESULTS' ANALYSIS AND THEORETICAL EVALUATION

This section provides simulations' results and theoretical analysis to prove the performance of the proposed PSO-MIMO detectors.

### 5.1. Experimental set-up

We evaluate the performance of proposed detectors for a $2\times 4$ ($N_t \times N_r$) MIMO-OFDM system with 4-QAM, 16-QAM, 32-QAM and 64-QAM constellations. In addition, the proposed detectors are also tested in a $4\times 4$ and $8\times 8$ 4-QAM MIMO configurations. One hundred and twenty-eight sub-carriers and cyclic prefix of length 32 are used. BER performance is analyzed at different SNR. The SNR ($E_b/N_o$) is the average signal-to-noise ratio per antenna ($P/\sigma_v^2$), where $\mathbf{P}$ is the average power per antenna and $\sigma_v^2$ is the noise variance. The simulation environment assumes Rayleigh flat-fading channel with no correlation between sub-channels. An average of no less than 30 000 simulations were taken to report statistically relevant results. In the simulated system, acceleration constants $\phi_1$ and $\phi_2$ are assumed to be unity for simplification, whereas $v_{max} = \pm 4$ for BPSO and $v_{max} = \pm 10$ for SPSO detection algorithms [9]. The particle size in SPSO and MPSO detection algorithms is kept as 16 for the result shown in Figure 5. Here random initial particle positions are assumed. For the BPSO-MIMO system as shown in Figures 7 and 8, particle size '$N_p$' depends upon the QAM size and the number of transmitters used in the MIMO system. In the case of BPSO, $N_p = b \times N_t$ where '$b$' is bits per symbol. For a $4\times 4$, 4-QAM system, '$N_p$' equals 8 and it grows to 16 for an $8\times 8$, 4-QAM system. Algorithm iteration '$N_{itr}$' is kept according to the system requirements. As an initial estimate we use the result of VBLAST for the results obtained in Figures 7 and 8. Lesser $N_{itr}$ will now be required due to refined initial solution guess instead of random particle positions as assumed in the earlier results obtained in Figures 5 and 6. For the results shown in Figures 7 and 8, '$N_{itr}$' is kept in the range of 10–20.

### 5.2. BER versus SNR performance

Figure 5 depicts BER versus $E_b/N_o$ performance of SPSO-MIMO and MPSO-MIMO detectors in comparison with the optimal ML. The former shows 4-dB, whereas the latter 2-dB reduced



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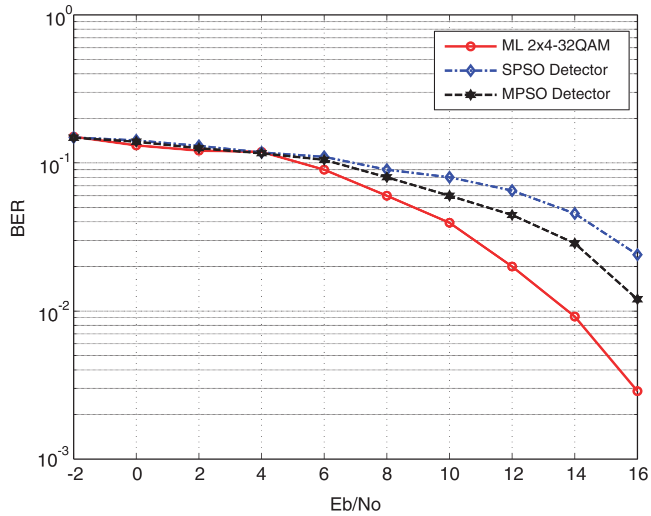


Figure 5. BER versus $E_b/N_o$ $2\times 4$ MIMO system.

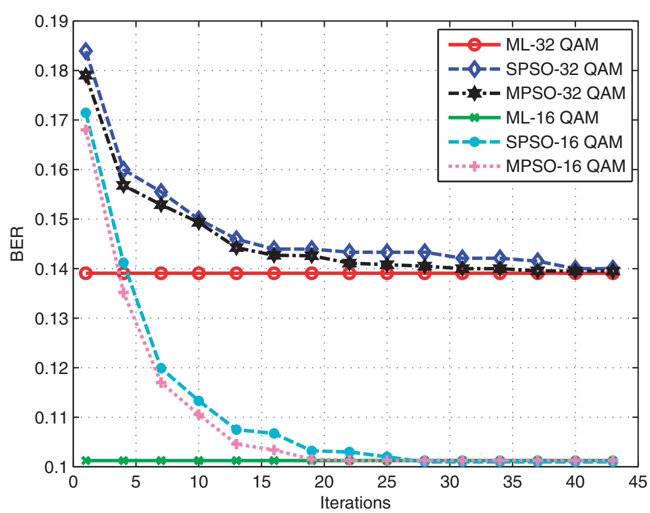


Figure 6. SPSO and MPSO algorithms convergence with iterations.

performance at $10^{-2}$ BER as compared with the ML detector. Figure 6 demonstrates the convergence behavior of SPSO and MPSO algorithms with increase in algorithm iterations using a random initial solution guess. SPSO-MIMO algorithm with 16-QAM converges to optimal BER in 25 iterations, whereas 32-QAM system takes 32 iterations to converge. Similarly, MIMO-MPSO detection requires 18 and 25 iterations with 16-QAM and 32-QAM systems to converge to the ML performance.



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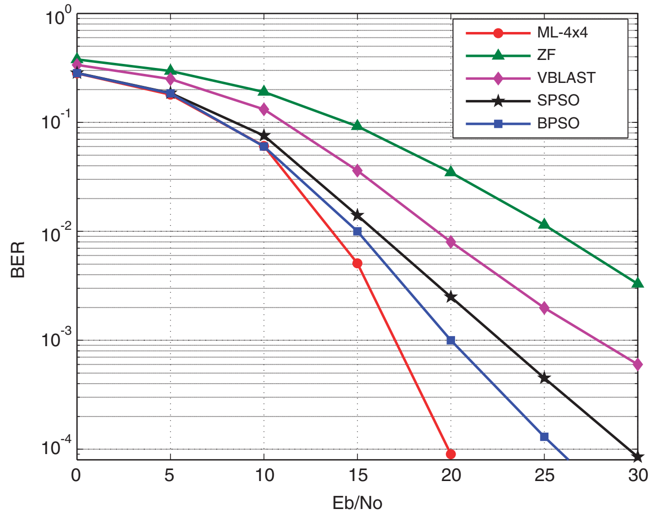


Figure 7. BER versus $E_{\mathrm{b}}/N_{\mathrm{o}}$ for 4-QAM $4\times4$ MIMO system.

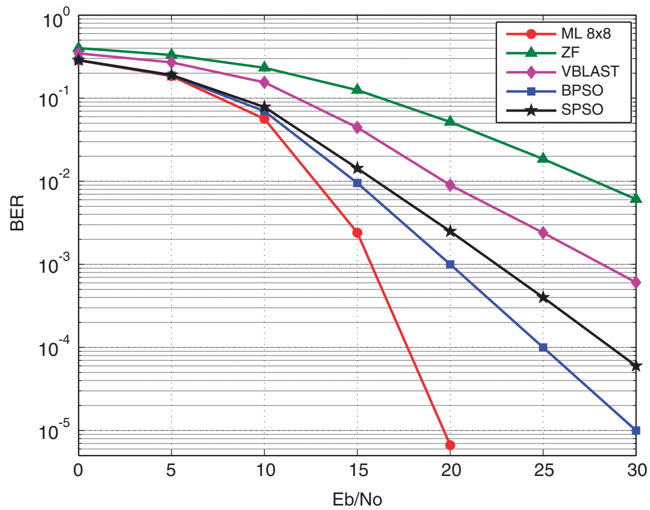


Figure 8. BER versus $E_{\mathrm{b}}/N_{\mathrm{o}}$ for 4-QAM $8\times8$ MIMO system.

Figure 7 shows the BER versus $E_{\mathrm{b}}/N_{\mathrm{o}}$ performance of BPSO and SPSO detectors compared with ML for $4\times4$ 4-QAM MIMO system. Now the initial solution guess of VBLAST is assumed for fast convergence. $N_{\mathrm{itr}}$ is kept at 10. At $10^{-3}$ BER, BPSO and SPSO detector results in 3- and 6-dB degraded BER performance with respect to ML. Similarly, for an $8\times8$, 4-QAM, MIMO system in Figure 8, at $10^{-3}$ BER, BPSO and SPSO algorithms result in 4- and 7-dB degraded BER performance in comparison with the optimal detector. However, a substantial ML complexity reduction is achieved, which is discussed in the next subsection.



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### 5.3. *Computational complexity theoretical evaluation*

Here we examine the computational complexity of the reported PSO-MIMO detectors and formulate a theoretical expression for computational complexity. A comparison with the conventional ML optimal detection method is also drawn. As the hardware cost of each algorithm is implementation-specific, we try to provide a rough estimate of complexity in terms of number of complex multiplications. The computational complexity is computed in terms of the $N_t$, $N_r$ and the constellation size $M$.

For the ML detector as seen from (5) $M^{N_t}(N_r N_t)$ multiplications are required for matrix multiplication operation and additional $M^{N_t} N_r$ multiplications are needed for square operation. Therefore, ML complexity becomes

$$\gamma_{ML} = N_r(N_t + 1)M^{N_t} \tag{11}$$

In the case of ZF detection, the pseudo-inverse of matrix $(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$ takes $4N_t^3 + 2N_t^2 N_r$ multiplications [34]. Therefore, ZF complexity becomes

$$\gamma_{ZF} = 4N_t^3 + 2N_t^2 N_r \tag{12}$$

For VBLAST the pseudo-inverse matrix is calculated $N_t$ times with decreasing dimension. In addition, the complexity of ordering and interference canceling is $\sum_{i=0}^{N_t-1}[N_t(N_t - i) + 2N_t]$. Therefore, total complexity of VBLAST ($\gamma_{VBLAST}$) results in

$$\gamma_{VBLAST} = \sum_{i=0}^{N_t}(4i^3 + 2N_r i^2) + \sum_{i=1}^{N_t-1}[N_t(N_t - i) + 2N_t] \tag{13}$$

$$= N_t^4 + (5/2 + 2/3N_r)N_t^3 + (7/2 + N_r)N_t^2 + 1/3N_t N_r \tag{14}$$

For the proposed detector, first fitness of each particle in population $N_p$ using (5) is calculated. Multiplication complexity ($\gamma_{PSO}$) becomes

$$\gamma_{PSO} = N_p(N_t N_r) \tag{15}$$

Velocity update in PSO and pheromone updates require $\mu_{vel}$ additional multiplications per iteration from (7). To reduce some complexity $w = 1$ and $\varphi_1 = \varphi_2 = 1$ is assumed. Therefore, $\mu_{vel}$ becomes 2 and the complexity becomes

$$\gamma_{PSO} = N_p(N_t N_r + \mu_{vel}) \tag{16}$$

This procedure is repeated $N_{itr}$ times to converge to the near-optimal BER performance. Therefore,

$$\gamma_{PSO} = N_p(N_t N_r + \mu_{vel})N_{itr} \tag{17}$$

The computational complexity for LS MPSO results in

$$\gamma_{MPSO} = N_p(N_t N_r + \mu_{vel})N_{itr}2^{b_s} \tag{18}$$

where $b_s$ represents the degree of LS in bits.

If the detectors take initial solution guess of ZF or VBLAST solution, its complexity is also added to get the resultant complexity $\gamma_{PSO\text{-}total}$.

$$\gamma_{PSO\text{-}total} = \gamma_{PSO} + (\gamma_{VBLAST} \text{ or } \gamma_{ZF}) \tag{19}$$



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Table II. Computational complexity comparison—MQAM 2 × 4-MIMO system.

| Method | 16-QAM | 32-QAM | 64-QAM |
|---|---|---|---|
| ML detector | 3072 | 12 288 | 49 152 |
| SPSO-MIMO using (15) | ($N_p = 10$, $N_{itr} = 25$, $\mu_{vel} = 2$) 2500 | ($N_p = 12$, $N_{itr} = 32$, $\mu_{vel} = 2$) 3480 | ($N_p = 16$, $N_{itr} = 38$, $\mu_{vel} = 2$) 6080 |
| ML complexity reduction $(\gamma_{ML} - \gamma_{PSO})/\gamma_{ML}$ | 19% | 71% | 88% |
| MPSO-MIMO using (16) | ($N_p = 10$, $N_{itr} = 18$, $\mu_{vel} = 2, b_s = 1$) 3600 | ($N_p = 12$, $N_{itr} = 25$, $\mu_{vel} = 2, b_s = 1$) 6000 | ($N_p = 16$, $N_{itr} = 32$, $\mu_{vel} = 2, b_s = 1$) 10 240 |
| ML complexity reduction | 14% (more complex) | 51% | 79% |

Table III. Computational complexity comparison—4QAM $N_t \times N_r$ MIMO system.

| Method | 4 × 4 | 8 × 8 |
|---|---|---|
| ML | 5120 | 4.7 M |
| SPSO-MIMO using (19) | ($N_p = 10$, $N_{itr} = 10$, $\mu_{vel} = 2, \gamma_{VBLAST} = 712$) 2512 | ($N_p = 20$, $N_{itr} = 20$, $\mu_{vel} = 2, \gamma_{VBLAST} = 8864$) 36 064 |
| ML complexity reduction $(\gamma_{ML} - \gamma_{PSO})/\gamma_{ML}$ | 51% | 99% |
| BPSO-MIMO using (19) | ($N_p = 8$, $N_{itr} = 10$, $\mu_{vel} = 2, \gamma_{VBLAST} = 712$) 2152 | ($N_p = 16$, $N_{itr} = 20$, $\mu_{vel} = 2, \gamma_{VBLAST} = 8864$) 29 984 |
| ML complexity reduction | 58% | 99% |

From (11) it is obvious that the complexity of ML is exponential with $N_t$ and $M$. ML complexity for a 4-QAM 4 × 4 system is 5120 and it grows to 4.7 M for the 8 × 8 system. This increase is even significant with higher-order modulation schemes in MIMO systems with more transmitters.

A detailed complexity comparison is shown in Tables II and III. However, this complexity estimate is only meaningful in the order-of-magnitude sense since it is based on the number of complex multiplications only. The above complexity is estimated on subcarrier-by-subcarrier for the MIMO-OFDM system.

### 5.4. *BER performance-computational complexity trade-off*

Table IV suggests that a reasonable performance-complexity trade-off exists when a comparison of the proposed detectors is drawn with the exhaustive search ML detector. With 32-QAM, MIMO-SPSO detectors improve the computation time by 71% and this improvement reaches 88% with 64-QAM. Similarly, 64-QAM, MIMO-MPSO detection algorithm reduces the computation time by 79% approximately. For a 4 × 4, 4-QAM system, at $10^{-3}$ BER the performance of BPSO detector is 3-dB lesser than ML with 58% ML complexity reduction. Similarly, in 8 × 8, 4-QAM system, the BPSO algorithm achieves $10^{-3}$ BER at 4-dB more SNR than ML, whereas ML complexity reduction is 99%.

### 5.5. *Effects of change in algorithm parameters and increase in iterations*

These detectors converge to near-optimal performance iteratively; however, these algorithms also experience saturation after reaching a particular threshold BER. Therefore, iteration tuning is



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Table IV. Performance complexity trade-off.

| Performance complexity comparison | $2\times4$ 32-QAM | $4\times4$ 4-QAM | $8\times8$ 4-QAM |
|---|---|---|---|
| ML and BPSO detector | | | |
| Complexity reduction | — | 58% | 99% |
| Performance degradation at $10^{-3}$ BER | — | 3 dB | 4 dB |
| ML and SPSO detector | | | |
| Complexity reduction | 71% | 51% | 99% |
| Performance degradation at $10^{-2}$ BER | 4 dB | 6 dB | 7 dB |
| ML and MPSO detector | | | |
| Complexity reduction | 51% | — | — |
| Performance degradation at $10^{-2}$ BER | 2 dB | — | — |

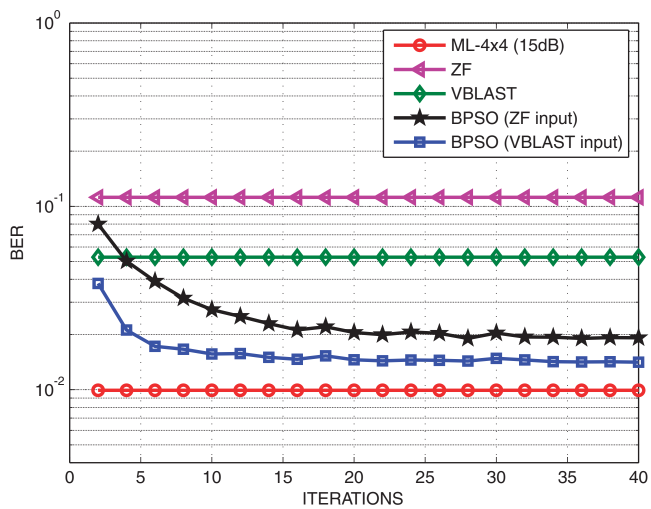


Figure 9. Convergence with iterations at 15-dB.

required for optimum performance. Figure 6 shows the convergence of SPSO and MPSO algorithms with an increase in iterations with random initialization. $N_{\mathrm{itr}}$ is kept at 18 and 25 for the 16-QAM and 32-QAM systems SPSO and MPSO systems. Figure 9 presents the convergence pattern of BPSO with ZF and VBLAST initial inputs. The algorithm gets saturated at around 10 iterations with VBLAST input and 15 iterations for the ZF initialization case. Therefore, $N_{\mathrm{itr}}$ is kept at 10 for a $4\times4$ MIMO system. The choice of good initial guess has an effect on the detectors convergence as can be seen from Figure 9.

Figure 10 shows the effect of changing the algorithm parameters on the detectors' performance. Values of the cognitive component ($c_1$) and social component ($c_2$) are changed. Results in Figure 10 assume $c_1=c_2=0.5, 1.49$ and 2. Larger values of social and cognitive components result in an improvement in BER performance. A possible reason is frequent fly over and coming back to a better solution is achieved with higher $c_1$ and $c_2$ values.



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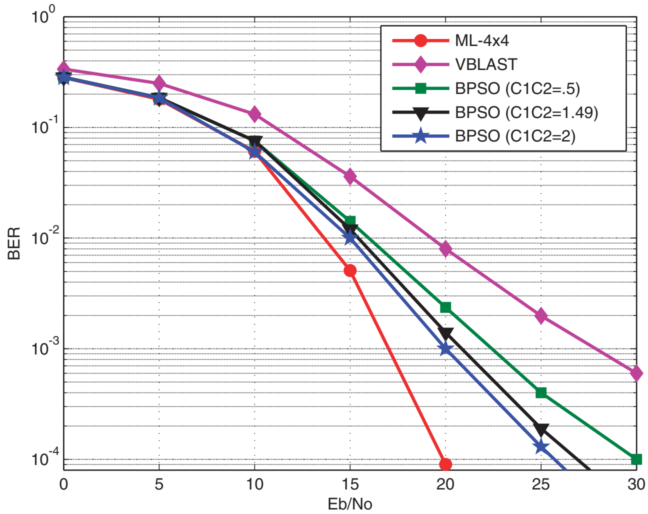


Figure 10. Effect on BER performance with change in social and cognitive components.

### *5.6. Comparison of different PSO-MIMO detection algorithms*

In this paper, we applied different PSO algorithms such as SPSO, MPSO and BPSO to optimize the exhaustive-search ML detection problem in a MIMO communications system. The BPSO-MIMO detection algorithm performs the best among the other PSO-MIMO techniques. The reason for efficient performance of the BPSO-MIMO detection algorithm is the inherent binary nature of the MIMO detection problem. The MIMO detection fitness function in (6) is best optimized using the BPSO algorithm. That is why the binary version of PSO has outperformed the other two MPSO-MIMO and SPSO-MIMO detection algorithms in BER performance as well as computational complexity.

### *5.7. An analysis of PSO algorithm as a MIMO detection technique*

Particle SI-assisted detection approach shows promising results. Their simple mathematical model, lesser implementation complexity, resistance to being trapped in local minima and guaranteed convergence to reasonable solution in lesser iterations make these nature inspired techniques a suitable candidate for real-time symbol detection in the MIMO system. PSO algorithms imitate nature's own ingenious ways to explore the search space to find out an optimal solution from a complex ML cost surface. The efficiency of these algorithms also lies in a simple computer code in the central algorithm with few parameters to tune. Exploratory-exploitive search approach, which is an essence of PSO, makes it an efficient ML search optimizer. The reduction in computation time with higher-order modulations and more transmitting antennas make these proposed detection algorithms particularly useful for high data rate communications system.

## 6. CONCLUSIONS

In this paper the application of PSO algorithms for symbol detection in a spatial multiplexing system was presented. These SI meta-heuristics proved to be powerful ML function optimizers.



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Their simple model with lesser implementation complexity makes them suitable for this NP-hard MIMO detection problem. PSO-optimized MIMO symbol detection methods approach near-optimal performance with significantly reduced computational complexity, especially for higher constellation systems with multiple transmitting antennas and larger constellation alphabet sizes, where conventional ML detector is computationally expensive and non-practical to implement. The simulation results suggest that the proposed PSO detectors reduce the ML computational complexity by as high as 99% with near-optimal BER performance for an $8\times 8$ MIMO-OFDM system. Therefore, these proposed detection algorithms are particularly suitable for future high-speed wireless communications system employing multiple antennas and higher-order modulation schemes.

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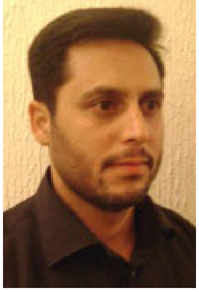

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