Particle Swarm Optimization

An introduction and its recent developments

A tutorial prepared for SEAL'06

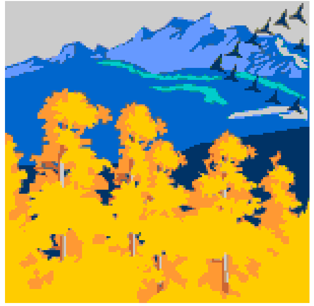
Xiaodong Li, School of Computer Science and IT, RMIT University, Melbourne, Australia

Outline

- Swarm Intelligence
- Introduction to PSO
  - PSO real-world applications
  - PSO variants
  - Communication topologies
- Speciation and niching methods in PSO
- PSO for optimization in dynamic environments
- PSO for multiobjective optimization

Swarm Intelligence

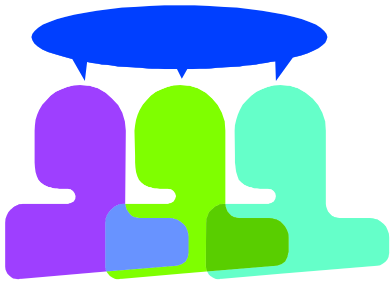
Swarm Intelligence



Swarm intelligence (SI) is an artificial intelligence technique based around the study of collective behavior in decentralized, self-organized systems.

SI systems are typically made up of a population of simple agents interacting locally with one another and with their environment. Although there is normally no centralized control structure dictating how individual agents should behave, local interactions between such agents often lead to the emergence of global behavior. Examples of systems like this can be found in nature, including ant colonies, bird flocking, animal herding, bacteria molding and fish schooling (from *Wikipedia*).

Swarm Intelligence



Mind is social…

Human intelligence results from social interaction:
Evaluating, comparing, and imitating one another, learning from experience and emulating the successful behaviours of others, people are able to adapt to complex environments through the discovery of relatively optimal patterns of attitudes, beliefs, and behaviours. (Kennedy & Eberhart, 2001).

Culture and cognition are inseparable consequences of human sociality:
Culture emerges as individuals become more similar through mutual social learning. The sweep of culture moves individuals toward more adaptive patterns of thought and behaviour.

Swarm Intelligence

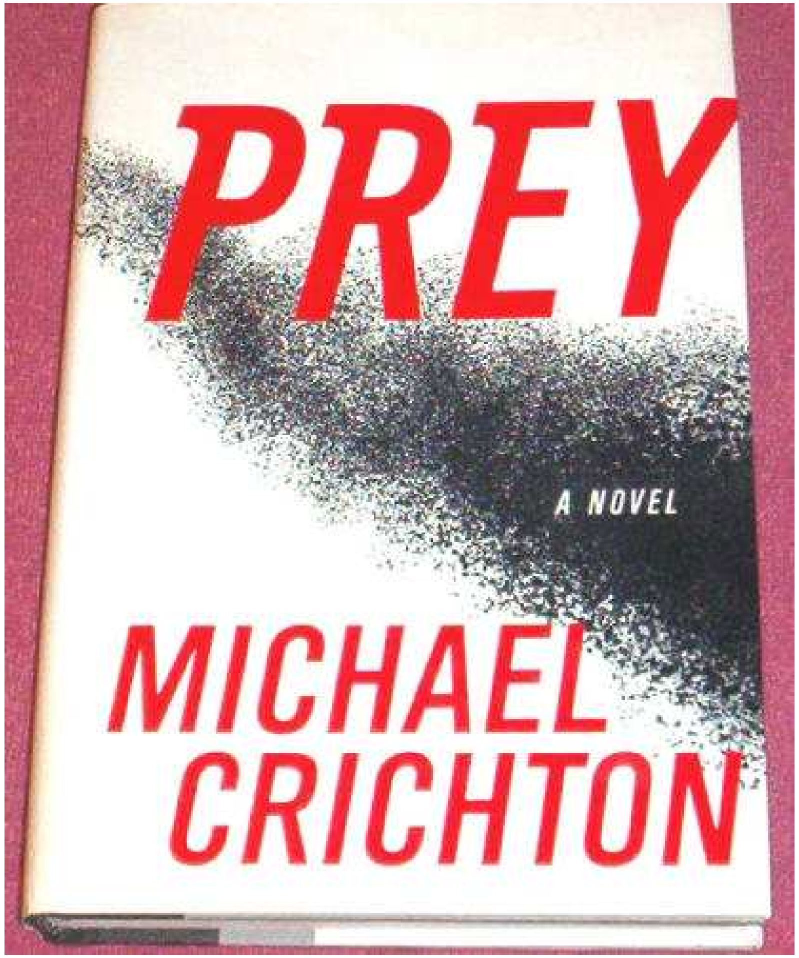
To model human intelligence, we should model individuals in a social context, interacting with one another.

Swarm Intelligence applications

- **Swarm-bots,** an EU project led by Marco Dorigo, aimed to study new approaches to the design and implementation of self-organizing and self-assembling artifacts (http://www.swarm-bots.org/).
- A 1992 paper by M. Anthony Lewis and George A. Bekey discusses the possibility of using swarm intelligence to control **nanobots** within the body for the purpose of killing cancer tumors.
- Artists are using swarm technology as a means of creating complex interactive environments.
  - Disney's ***The Lion King*** was the first movie to make use of swarm technology (the stampede of the bisons scene).
  - The movie "***Lord of the Rings***" has also made use of similar technology during battle scenes.

(Some examples from *Wikipedia*)

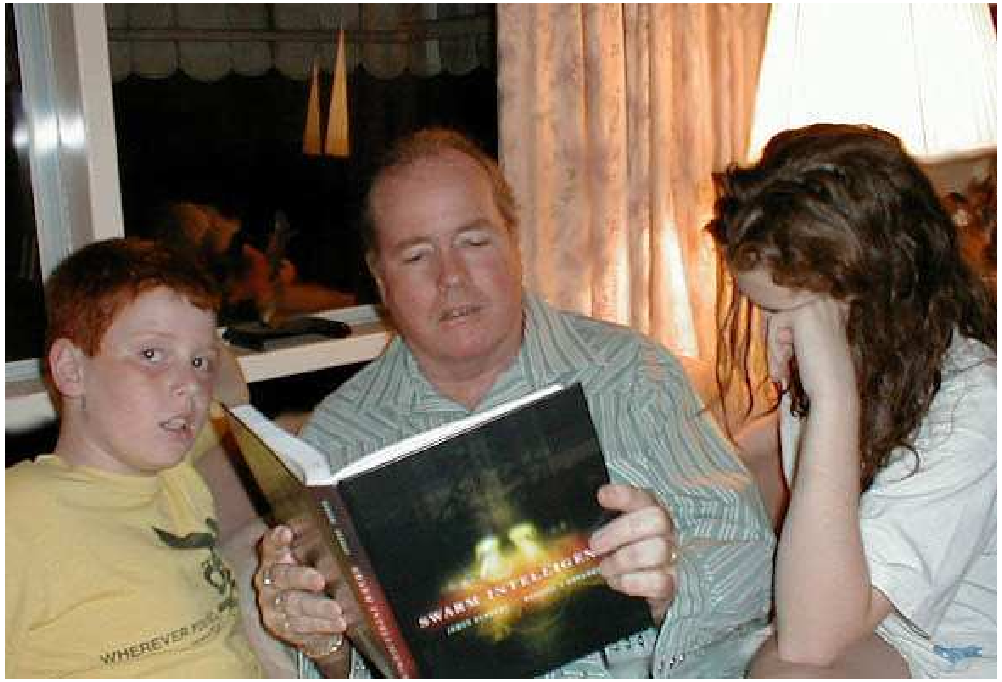
Novel about swarm



"… *Within hours of his arrival at the remote testing center, Jack discovers his wife's firm has created self-replicating nanotechnology--a literal swarm of microscopic machines. Originally meant to serve as a military eye in the sky, the swarm has now escaped into the environment and is seemingly intent on killing the scientists trapped in the facility*." (Michael Crichton, 2002)

Particle Swarm Optimization

The inventors:



James Kennedy

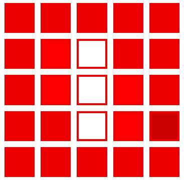
Russell Eberhart

Particle Swarm Optimization

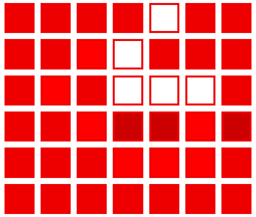
PSO has its roots in Artificial Life and social psychology, as well as engineering and computer science.

The particle swarms in some way are closely related to cellular automata (CA):

a) individual cell updates are done in parallel
b) each new cell value depends only on the old values of the cell and its neighbours, and
c) all cells are updated using the same rules (Rucker, 1999).



Blinker

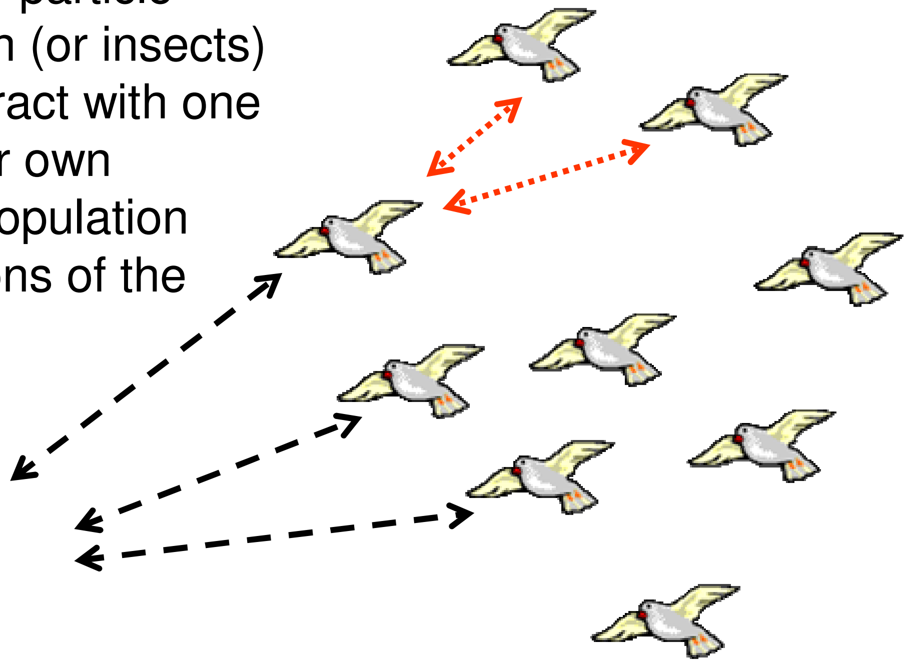


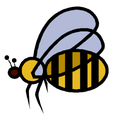
Glider

Individuals in a particle swarm can be conceptualized as cells in a CA, whose states change in many dimensions simultaneously.

Particle Swarm Optimization

As described by the inventers James Kennedy and Russell Eberhart, "particle swarm algorithm imitates human (or insects) social behavior. Individuals interact with one another while learning from their own experience, and gradually the population members move into better regions of the problem space".

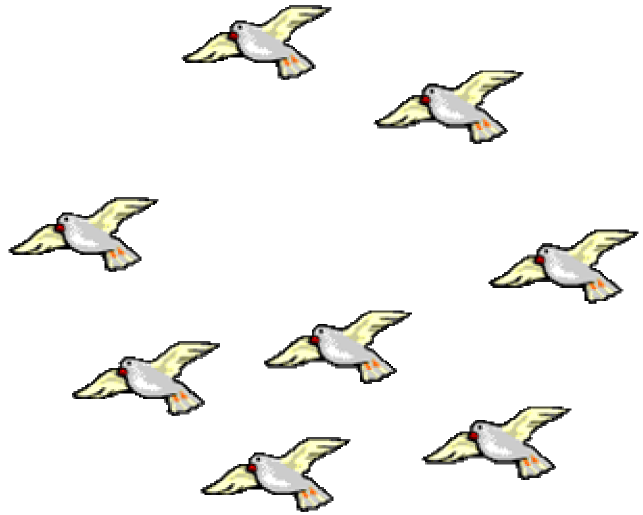


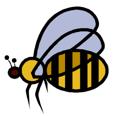


Why named as "Particle", not "points"? Both Kennedy and Eberhart felt that velocities and accelerations are more appropriately applied to particles.

Particle Swarm Optimization

As described by the inventers James Kennedy and Russell Eberhart, “particle swarm algorithm imitates human (or insects) social behavior. Individuals interact with one another while learning from their own experience, and gradually the population members move into better regions of the problem space”.





Why named as “Particle”, not “points”? Both Kennedy and Eberhart felt that velocities and accelerations are more appropriately applied to particles.

PSO Precursors

Reynolds (1987)'s simulation **Boids** – a simple flocking model consists of three simple local rules:

- **Collision avoidance**: pull away before they crash into one another;
- **Velocity matching**: try to go about the same speed as their neighbours in the flock;
- **Flock centering**: try to move toward the center of the flock as they perceive it.

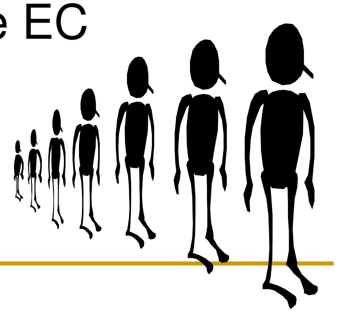
A demo: http://www.red3d.com/cwr/boids/
With just the above 3 rules, **Boids** show very realistic flocking behaviour.

Heppner (1990) interests in rules that enabled large numbers of birds to flock synchronously.

Its links to Evolutionary Computation

"*In theory at least, individual members of the school can profit from the discoveries and previous experience of all other members of the school during the search for food. This advantage can become decisive, outweighing the disadvantages of competition for food items, whenever the resource is unpredictably distributed in patches*" (by Sociobiologist E. O. Wilson)

- Both PSO and EC are population based.
- PSO also uses the fitness concept, but, less-fit particles do not die. No "survival of the fittest".
- No evolutionary operators such as crossover and mutation.
- Each particle (candidate solution) is varied according to its past experience and relationship with other particles in the population.
- Having said the above, there are hybrid PSOs, where some EC concepts are adopted, such as selection, mutation, etc.



PSO applications

Problems with continuous, discrete, or mixed search space, with multiple local minima.

- Evolving neural networks:
  - Human tumor analysis;
  - Computer numerically controlled milling optimization;
  - Battery pack state-of-charge estimation;
  - Real-time training of neural networks (Diabetes among Pima Indians);
  - Servomechanism (time series prediction optimizing a neural network);
- Reactive power and voltage control;
- Ingredient mix optimization;
- Pressure vessel (design a container of compressed air, with many constraints);
- Compression spring (cylindrical compression spring with certain mechanical characteristics);
- Moving Peaks (multiple peaks dynamic environment); and more

PSO can be tailor-designed to deal with specific real-world problems.

First PSO model

cognitive component

social component

velocity

$$\vec{v}_{t+1} = \vec{v}_t + \boxed{\vec{R}_1[0,\frac{\varphi}{2}] \otimes (\vec{p}_i - \vec{x}_t)} + \boxed{\vec{R}_2[0,\frac{\varphi}{2}] \otimes (\vec{p}_g - \vec{x}_t)}$$

$$\vec{x}_{t+1} = \vec{x}_t + \vec{v}_{t+1}$$

Velocity $\vec{v}_{t+1}$ (which denotes the amount of change) is a function of the difference between the individual's personal best $\vec{p}_i$ and its current position, and the difference between the neighborhood's best $\vec{p}_g$ and its current position (Kennedy & Eberhart, 2001). Note that the symbol $\otimes$ denotes a point-wise vector multiplication.

Since $\vec{\varphi}_1 = \vec{R}_1[0,\frac{\varphi}{2}]$ and $\vec{\varphi}_2 = \vec{R}_2[0,\frac{\varphi}{2}]$ give vectors of random numbers within a specified range, the particle will cycle unevenly around a point defined as the weighted average of $\vec{p}_i$ and $\vec{p}_g$:

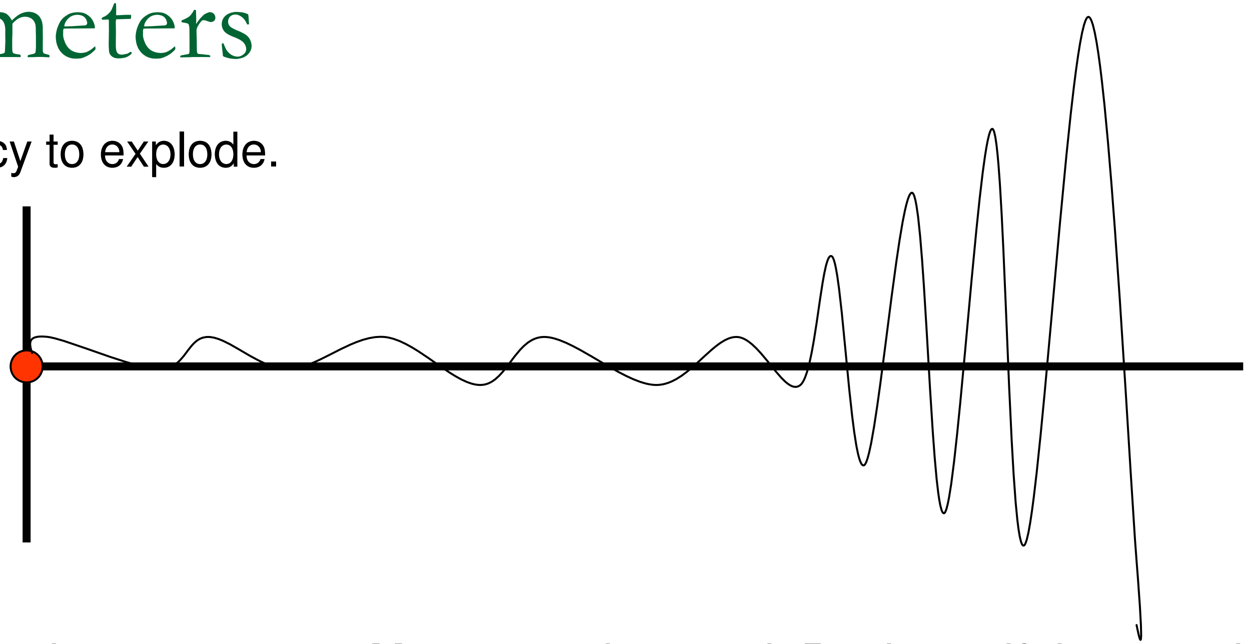
$$\frac{\vec{\varphi}_1 \otimes \vec{p}_i + \vec{\varphi}_2 \otimes \vec{p}_g}{\vec{\varphi}_1 + \vec{\varphi}_2}$$

Pseudocode of a basic PSO

```
Randomly generate an initial population
repeat
    for i = 1 to population_size do
        if f(x⃗_i) < f(p⃗_i) then p⃗_i = x⃗_i;
        p⃗_g = min(p⃗_neighbours);
        for d =1 to dimensions do
            velocity_update();
            position_update();
        end
    end
until termination criterion is met.
```

Parameters

Tendency to explode.



To prevent it, a parameter **Vmax** can be used. Basically if the velocity value exceeds ±**Vmax**, it gets reset to ±**Vmax** accordingly.

Control parameter $\varphi_{md} = \varphi_{1d} + \varphi_{2d}$ for the *d*-th dimension, called “acceleration constant”:

- if it is set too small, the trajectory of a particle falls and rises slowly;
- As its value is increased, the frequency of the particle oscillating around the weighted average of p_{id} and p_{gd} is also increased.

Inertia weight PSO

To further control the search, Shi and Eberhart (1998) proposed to use an “inertia weight” parameter:

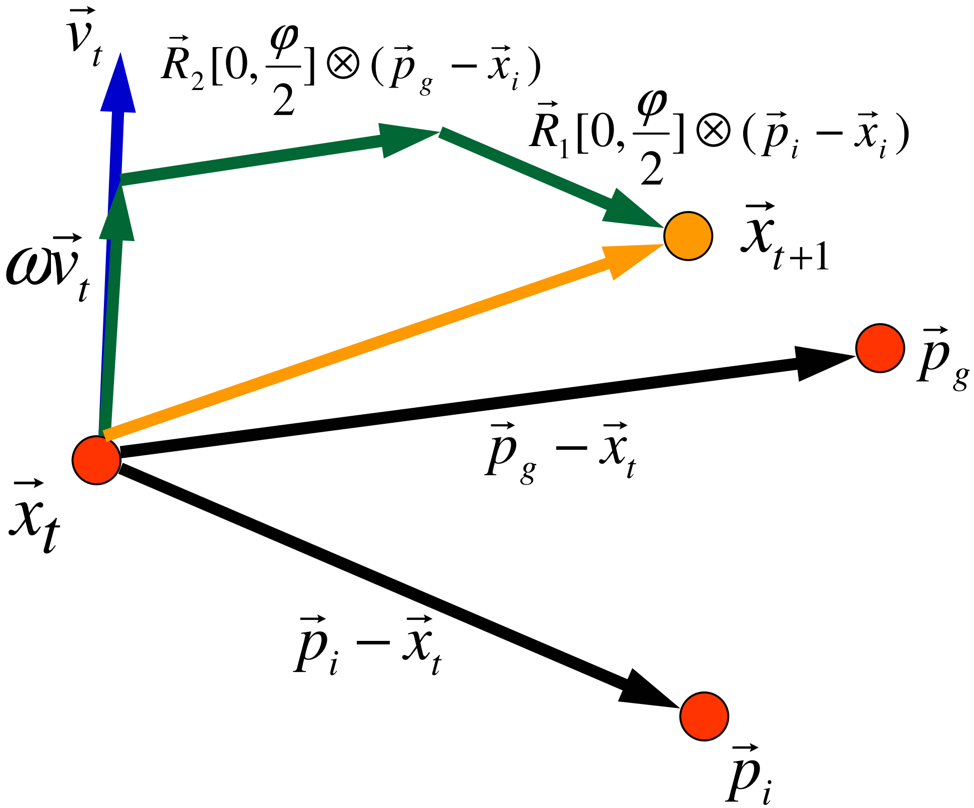
inertia weight

$$\vec{v}_{t+1} = W \; \vec{v}_t + \vec{R}_1[0, \frac{\varphi}{2}] \otimes (\vec{p}_i - \vec{x}_t) + \vec{R}_2[0, \frac{\varphi}{2}] \otimes (\vec{p}_g - \vec{x}_t)$$

$$\vec{x}_{t+1} = \vec{x}_t + \vec{v}_{t+1}$$

Eberhart and Shi suggested to use the inertia weight which decreasing over time, typically from 0.9 to 0.4, with $\frac{\varphi}{2} = 2.0$. It has the effect of narrowing the search, gradually changing from an exploratory to an exploitative mode.

Visualizing PSO



Constriction PSO

Clerc and Kennedy (2000) suggested a more generalized PSO, where a constriction coefficient (Type 1'' coefficient) is applied to both terms of the velocity formula. Clerc shows that the constriction PSO can converge without using **Vmax**:

constriction factor

$$\vec{v}_{t+1} = \boxed{\chi} \, (\vec{v}_t + \vec{R}_1[0, \frac{\varphi}{2}] \otimes (\vec{p}_i - \vec{x}_t) + \vec{R}_2[0, \frac{\varphi}{2}] \otimes (\vec{p}_g - \vec{x}_t))$$

$$\vec{x}_{t+1} = \vec{x}_t + \vec{v}_{t+1}$$

where $\frac{\varphi}{2}$ is a positive number, often set to 2.05; and the constriction factor χ set 0.7289 (Clerc and Kennedy 2002).

By using the constriction coefficient, the amplitude of the particle's oscillation decreases, resulting in its convergence over time.

Fully Informed PSO (FIPS)

The two terms in the constriction PSO are of the same form, hence can be condensed to the following (Mendes & Kennedy, 2004):

$$\vec{v}_{t+1} = \chi(\vec{v}_t + \vec{\varphi}_m \otimes (\vec{p}_m - \vec{x}_t))$$

$$\vec{x}_{t+1} = \vec{x}_t + \vec{v}_{t+1}$$

where $\vec{\varphi}_m = \vec{\varphi}_1 + \vec{\varphi}_2$ and $\vec{p}_m = (\vec{\varphi}_1 \otimes \vec{p}_i + \vec{\varphi}_2 \otimes \vec{p}_g)/(\vec{\varphi}_1 + \vec{\varphi}_2)$.

This shows that that a particle tends to converge towards a point determined by $\vec{p}_m$, which a weighted average of its previous best $\vec{p}_i$ and the neighbourhood's best $\vec{p}_g$. $\vec{p}_m$ can be further generalized to any number of terms:

$$\vec{v}_{t+1} = \chi(\vec{v}_t + \sum_{k \in N} \vec{R}[0, \frac{\varphi}{|N|}] \otimes (\vec{p}_k - \vec{x}_t))$$

N denotes the neighbourhood, and $\vec{p}_k$ the best previous position found by the *k*-th particle in *N*. If |*N*/ equals 2, then the above is a generalization of the canonical PSO.

Essential particle swarm(1)

Kennedy (2006) describes PSO in the following form:

New Position = Current Position +
Persistence +
Social Influence.

If we substitute $\vec{v}_t = \vec{x}_t - \vec{x}_{t-1}$ in FIPS, then we have:

$$\vec{x}_{t+1} = \vec{x}_t + \chi\left(\overbrace{\boxed{(\vec{x}_t - \vec{x}_{t-1})}}^{\text{Persistence}} + \overbrace{\boxed{\sum_{k \in N} \vec{R}[0, \frac{\varphi}{|N|}] \otimes (\vec{p}_k - \vec{x}_t)}}^{\text{Social influence}}\right)$$

Persistence indicates the tendency of a particle to persist in moving in the same direction it was moving previously.

Essential particle swarm(2)

The social influence term can be further expanded:

New Position = Current Position +
Persistence +
Social Central Tendency +
Social Dispersion

Social central tendency can be estimated, for example by taking the mean of previous bests relative to the particle's current position (still open-ended questions)

Social dispersion may be estimated by taking the distance of a particle's previous best to any neighbor's previous best; or by averaging pair-wise distances between the particle and some neighbors.

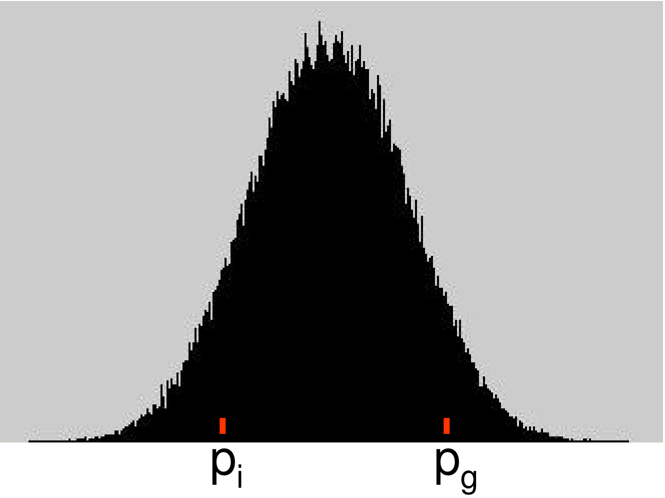
Some distributions such as Gaussian, double-exponential and Cauchy were used by Kennedy (2006).

Bare Bones PSO

What if we drop the velocity term? Is it necessary?

Kennedy (2003) carried out some experiments using a PSO variant, which drops the velocity term from the PSO equation.

If p_i and p_g were kept constant, a canonical PSO samples the search space following a bell shaped distribution centered exactly between the p_i and p_g.

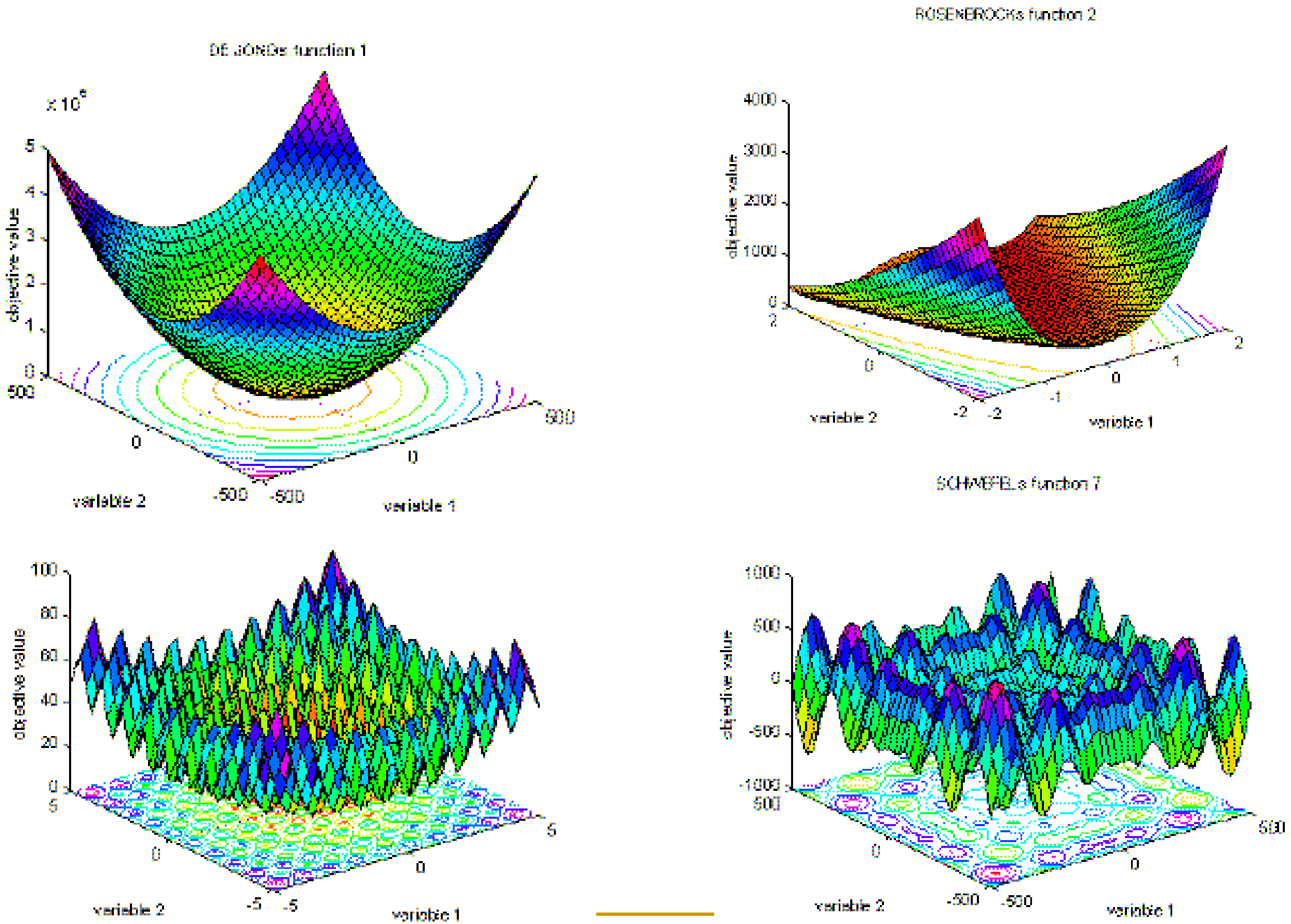


This bare bones PSO produces normally distributed random numbers around the mean $(p_{id} + p_{gd})/2$ (for each dimension *d*), with the standard deviation of the Gaussian distribution being $|p_{id} - p_{gd}|$.

Some PSO variants

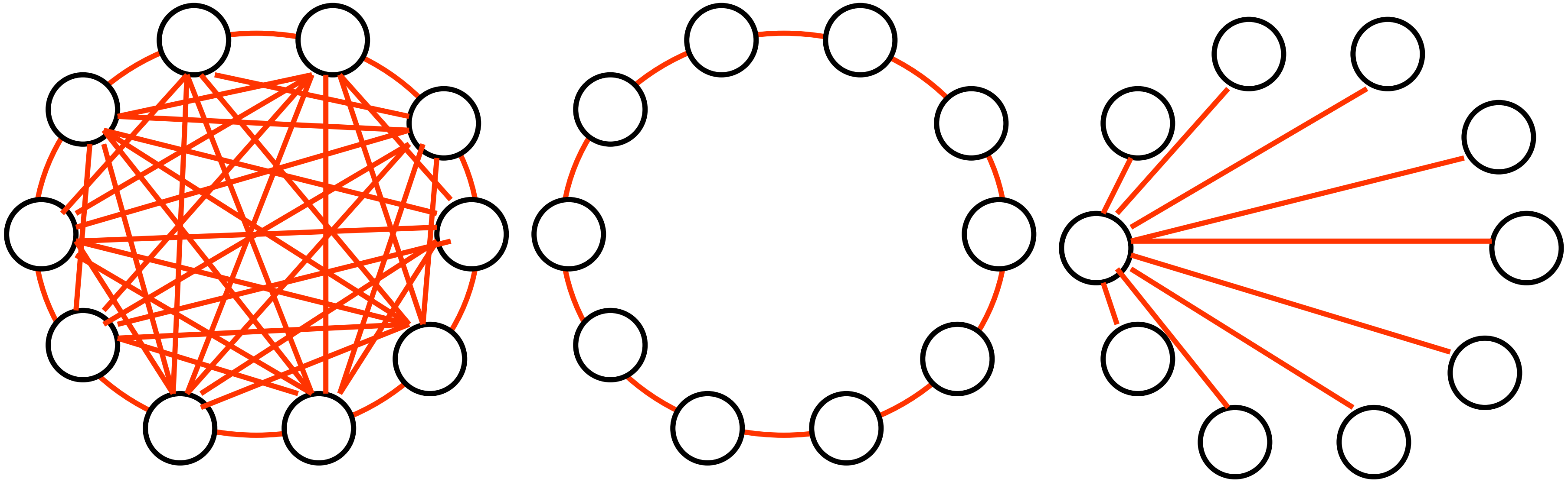
- Tribes (Clerc, 2006) – aims to adapt population size, so that it does not have to be set by the users;
- ARPSO (Riget and Vesterstorm, 2002) – uses a diversity measure to alternate between 2 phases;
- Dissipative PSO (Xie, et al., 2002) – increasing randomness;
- PSO with self-organized criticality (Lovbjerg and Krink, 2002) – aims to improve diversity;
- Self-organizing Hierachicl PSO (Ratnaweera, et al. 2004);
- FDR-PSO (Veeramachaneni, et al., 2003) – using nearest neighbour interactions;
- PSO with mutation (Higashi and Iba, 2003; Stacey, et al., 2004)
- Cooperative PSO (van den Bergh and Engelbrecht, 2005) – a cooperative approach
- DEPSO (Zhang and Xie, 2003) – aims to combine DE with PSO;
- CLPSO (Liang, et al., 2006) – incorporate learning from more previous best particles.

Test functions



Note: Demos on some test functions using a PSO.

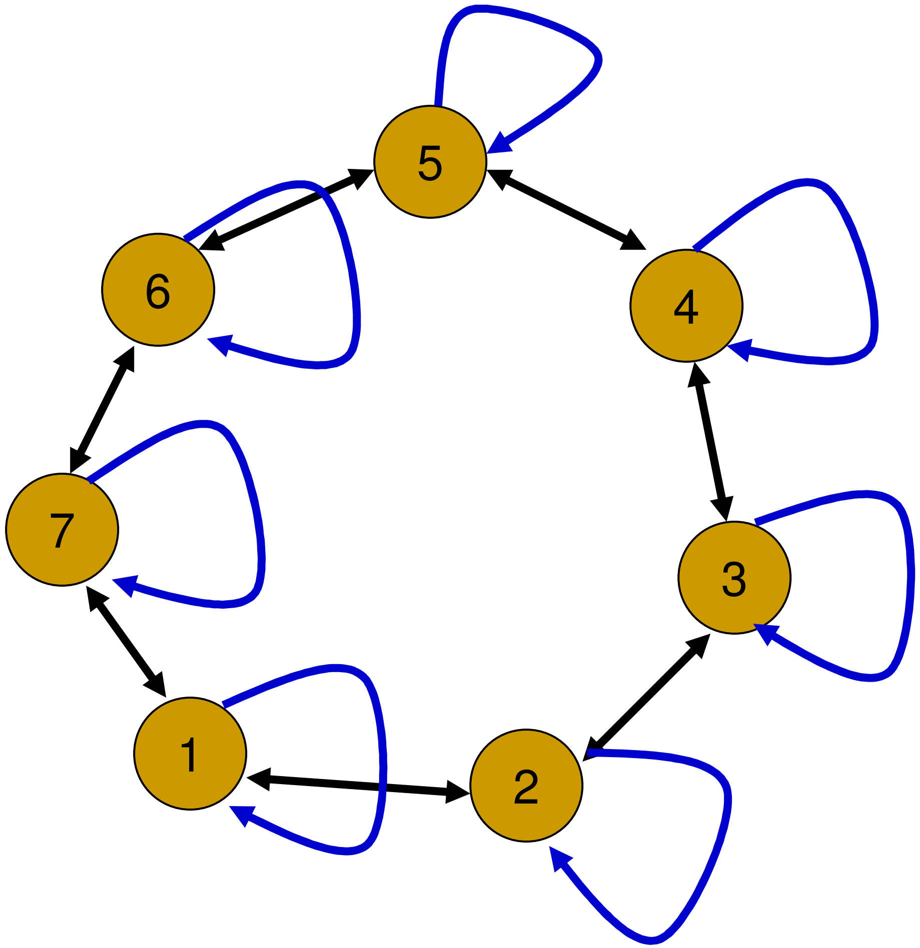
Communication topologies (1)



Two most common models:

- **gbest**: each particle is influenced by the best found from the entire swarm.
- **lbest**: each particle is influenced only by particles in local neighbourhood.

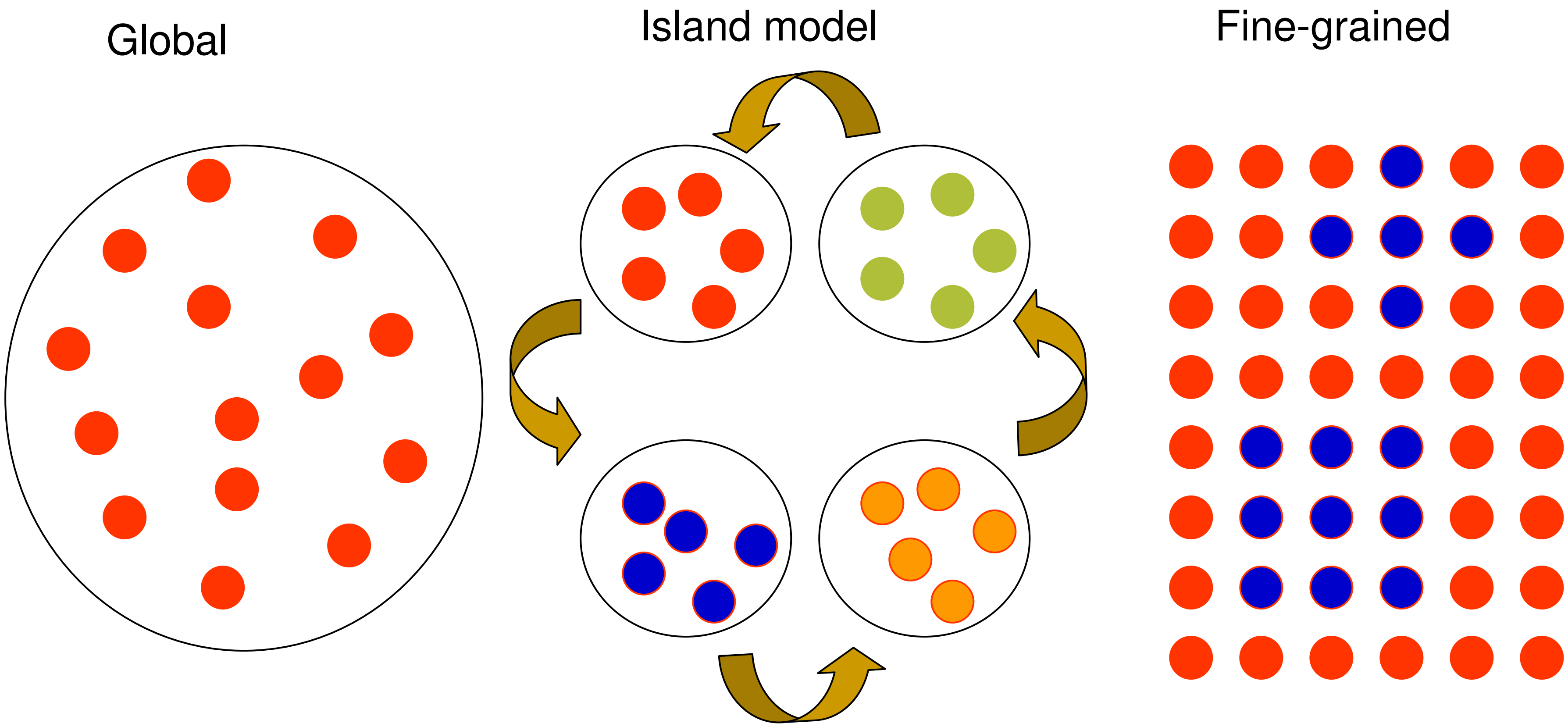
Communication topologies (2)



Graph of influence of a swarm of 7 particles. For each arc, the particle origin influence (informs) the end particle (Clerc, 2006)

This graph of influence can be also expanded to include previous best positions.

Communication topologies (3)



Communication topologies (4)

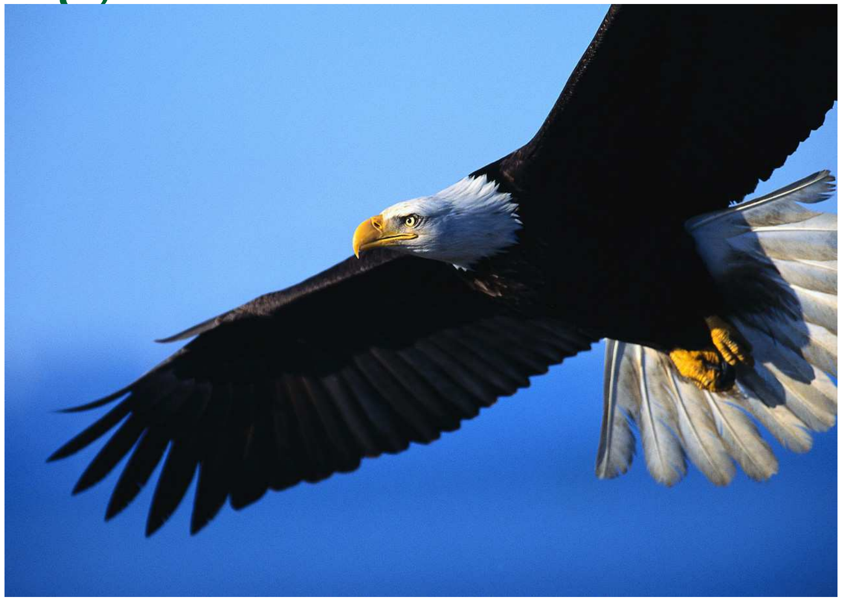
Which one to use?

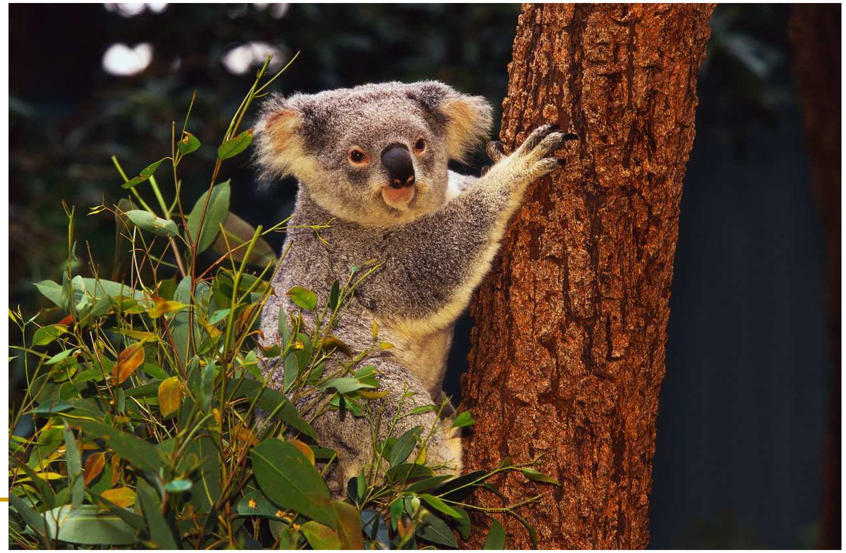
Balance between exploration and exploitation…

gbest model propagate information the fastest in the population; while the **lbest** model using a ring structure the slowest. For complex multimodal functions, propagating information the fastest might not be desirable. However, if this is too slow, then it might incur higher computational cost.

Mendes and Kennedy (2002) found that von Neumann topology (north, south, east and west, of each particle placed on a 2 dimensional lattice) seems to be an overall winner among many different communication topologies.

Speciation and niching





Speciation and niching

Biological species concept: *a species is a group of actually or potentially interbreeding individuals who are reproductively isolated from other such groups*.

The definition of a species is still debatable.

Most researchers believe either the **morphological** species concept (ie., members of a species look alike and can be distinguished from other species by their appearance), or the **biological** species concept (a species is a group of actually or potentially interbreeding individuals who are reproductively isolated from other such groups). Both definitions have their weaknesses.

Speciation and niching

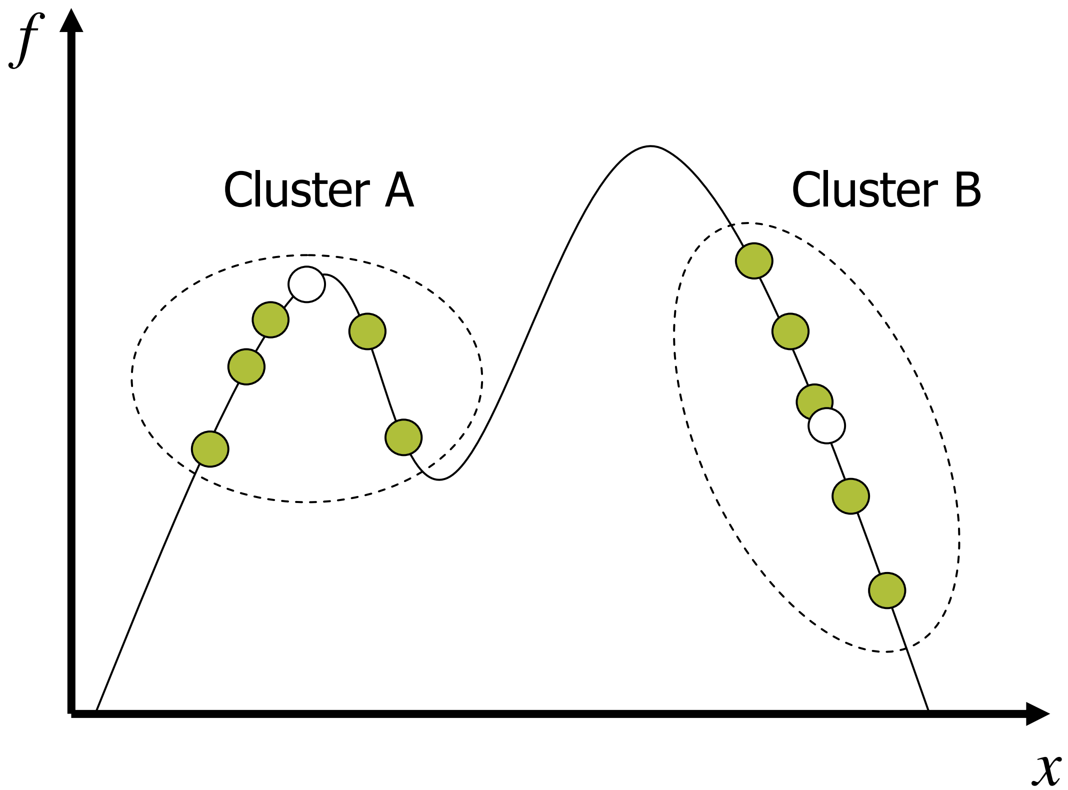
- Kennedy (2000) proposed a *k*-means clustering technique;
- Parsopoulos and Vrahitis (2001) used a stretching function;
- Brits et al. (2002) proposed a NichePSO;
- Many other niching methods developed for Evolutionary Algorithms, such as Crowding method, fitness-sharing, clearing, etc.
- Petrowski (1996) introduced a clearing procedure, and later on Li, et al. (2002) introduced a species conserving genetic algorithm (SCGA) for multimodal optimization.

The notion of species:

- A population is classified into groups according to their similarity measured by Euclidean distance.

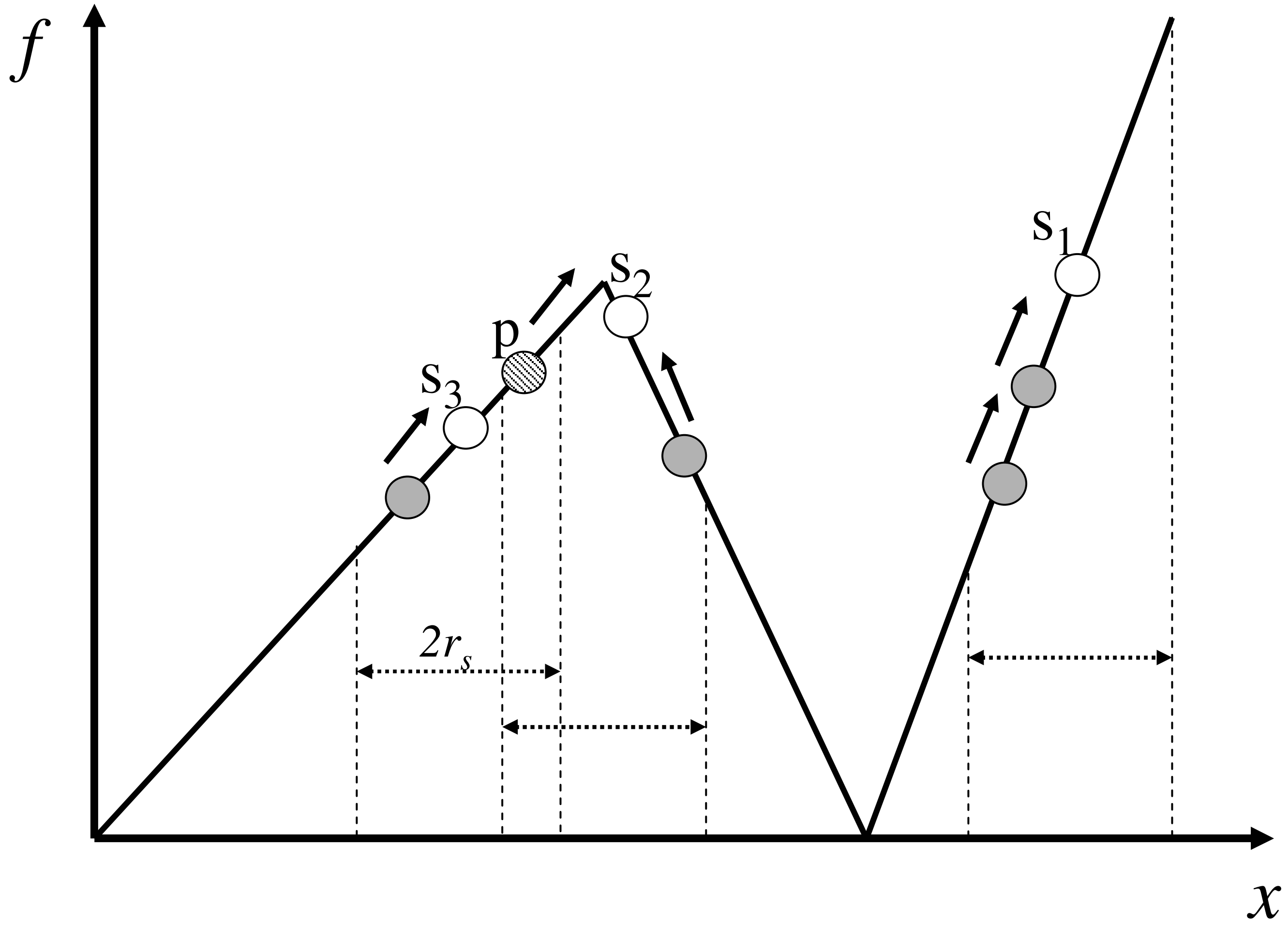
- The definition of a species also depends on another parameter r_s ,which denotes the radius measured in Euclidean distance from the center of the a species to its boundary.

Clustering-based PSO



Cluster A's center performs better than all members of cluster A, whereas cluster B's center performs better than some and worse than others.

Speciation-based PSO



An example of how to determine the species seeds from the population at each iteration. s_1, s_2, and s_3 are chosen as the species seeds. Note that p follows s_2.

Speciation-based PSO

Step 1: Generate an initial population with randomly generated particles;

Step 2: Evaluate all particle individuals in the population;

Step 3: Sort all particles in descending order of their fitness values (i.e., from the best-fit to least-fit ones);

Step 4: Determine the species seeds for the current population;

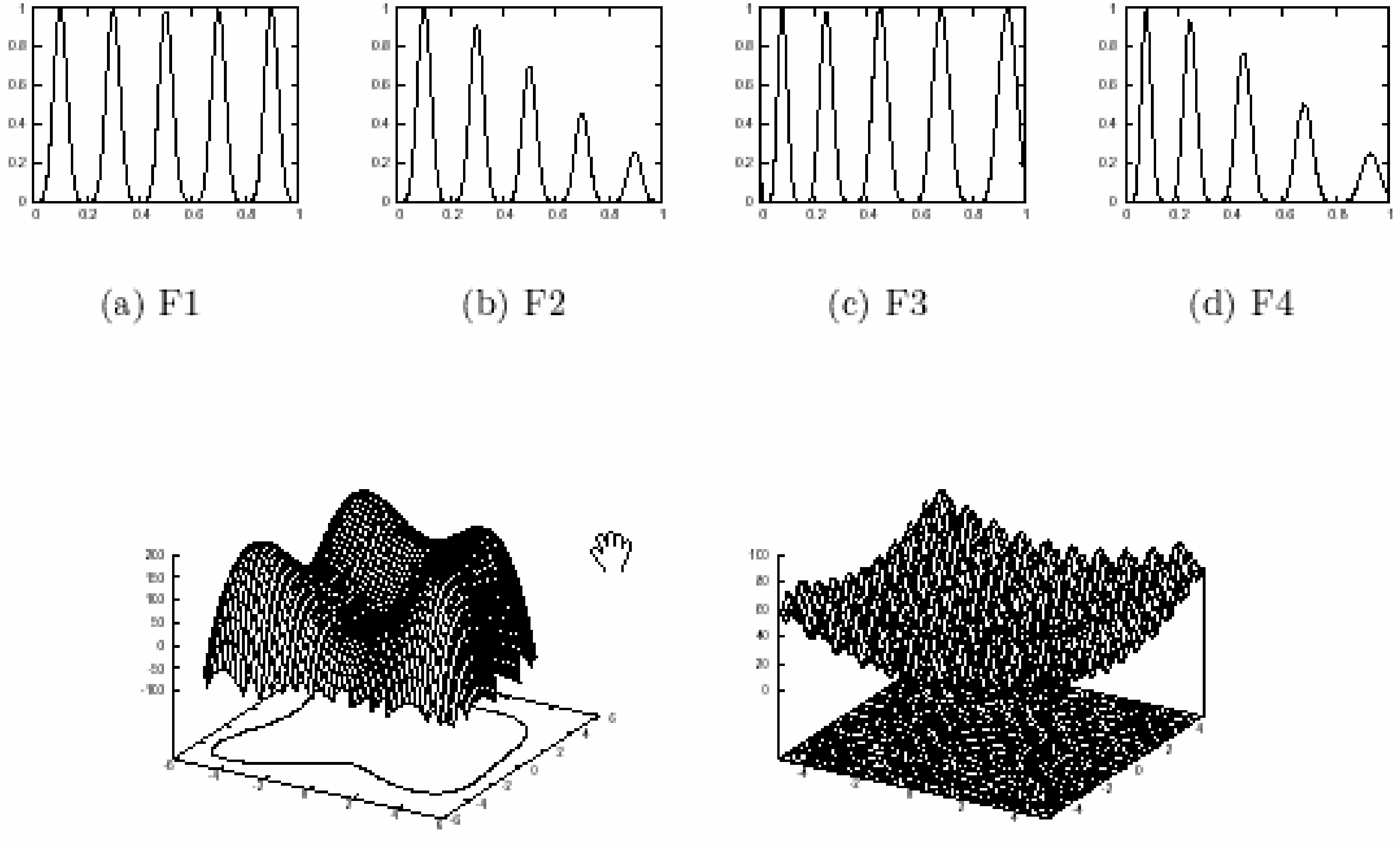
Step 5: Assign each species seed identified as the $\vec{p}_g$ to all individuals identified in the same species;

Step 6: Adjusting particle positions according to the PSO velocity and position update equation (1) and (2);

Step 7: Go back to step 2), unless termination condition is met.

Multimodal problems

Multimodal functions



(a) F1 (b) F2 (c) F3 (d) F4

(e) F5:Himmelblau's function (f) F6:Rastrigin function

Simulation runs

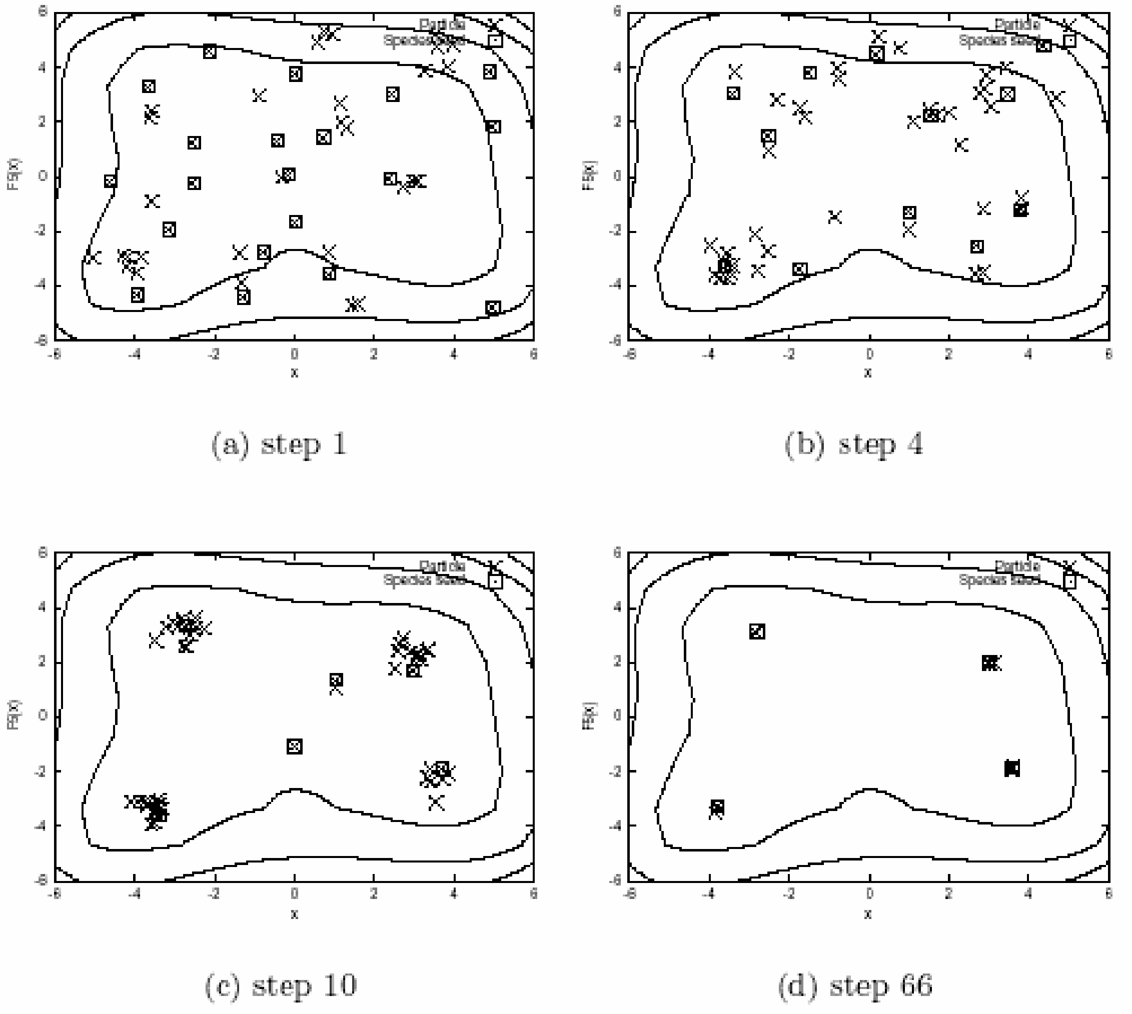
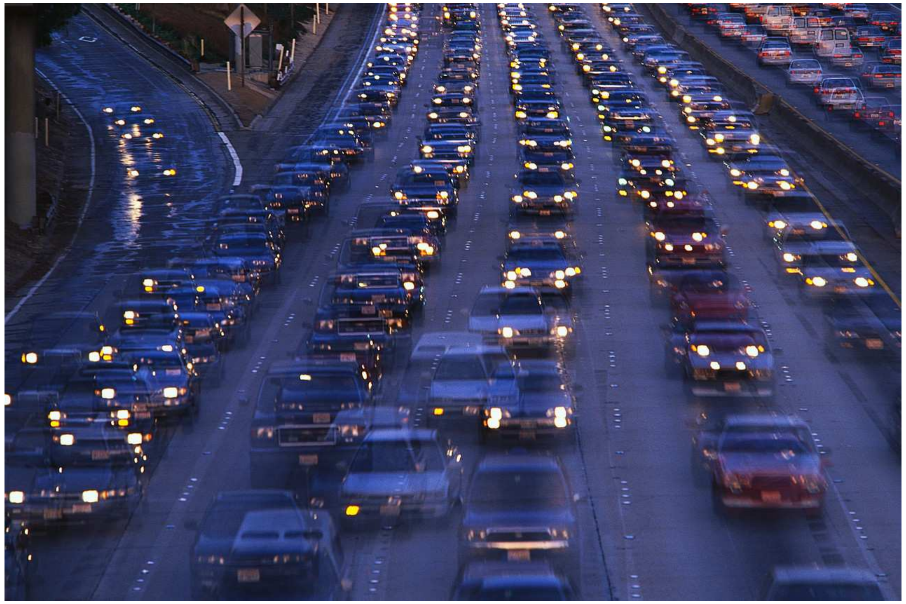


Fig. 6. A simulation run of SPSO on F5 - step 1, 4, 10 and 66.

Refer to Li (2004) for details.

Optimization in a dynamic environment

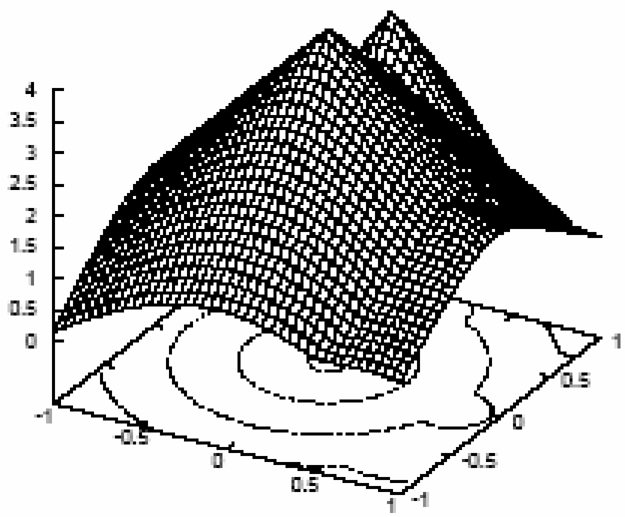
Many real-world optimization problems are dynamic and require optimization algorithms capable of adapting to the changing optima over time.

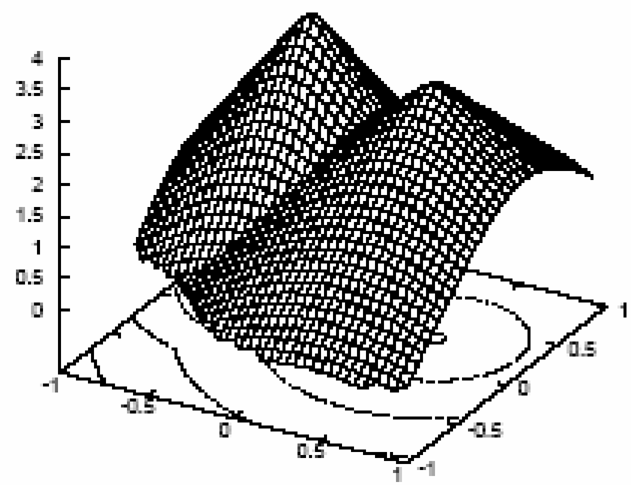


E.g., Traffic conditions in a city change dynamically and continuously. What might be regarded as an optimal route at one time might not be optimal in the next minute.

In contrast to optimization towards a static optimum, in a dynamic environment the goal is to track as closely as possible the dynamically changing optima.

Optimization in a dynamic environment





Three peak multimodal environment, before (above left) and after (above right) movement of optima. Note that the small peak to the right of the figure becomes hidden and that the highest point switches optimum (Parrott and Li, 2006).

Why PSO?

- With a population of candidate solutions, a PSO algorithm can maintain useful information about characteristics of the environment.
- PSO, as characterized by its fast convergence behaviour, has an in-built ability to adapt to a changing environment.
- Some early works on PSO have shown that PSO is effective for locating and tracking optima in both static and dynamic environments.

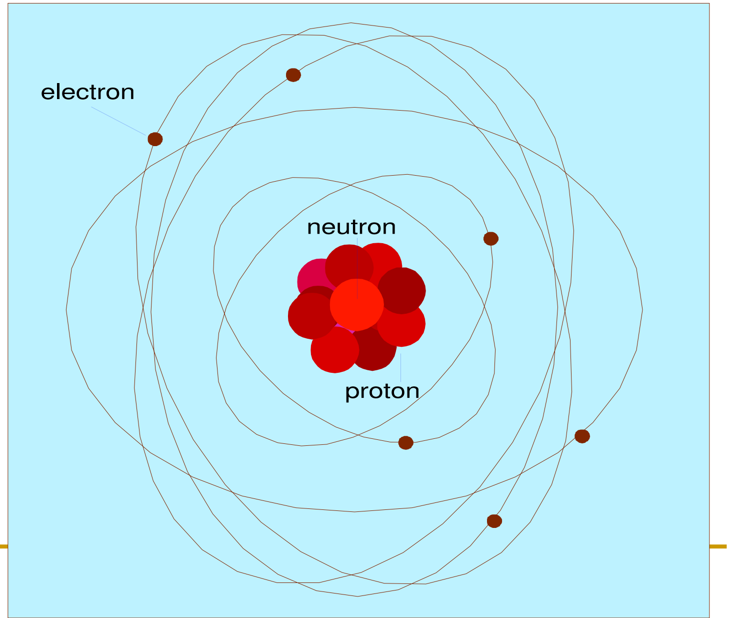
Two major issues must be resolved when dealing with dynamic problems:

- How to **detect** that a change in the environment has actually occurred?
- How to **respond** appropriately to the change so that the optima can still be tracked?

Related work

- Tracking the changing optimum of a unimodal parabolic function (Eberhart and Shi, 2001).
- Carlisle and Dozier (2002) used a randomly chosen sentry particle to detect if a change has occurred.
- Hu and Eberhart (2002) proposed to re-evaluate the global best particle and a second best particle.
- Carlisle and Dozier (2002) proposed to re-evaluate all personal bests of all particles when a change has been detected.
- Hu and Eberhart (2002) studied the effects of re-randomizing various proportions of the swarm.

- Blackwell and Bentley (2002) introduced charged swarms.
- Blackwell and Branke (2004, 2006) proposed an interacting multi-swarm PSO (using quantum particles) as a further improvement to the charged swarms.



Set the scope

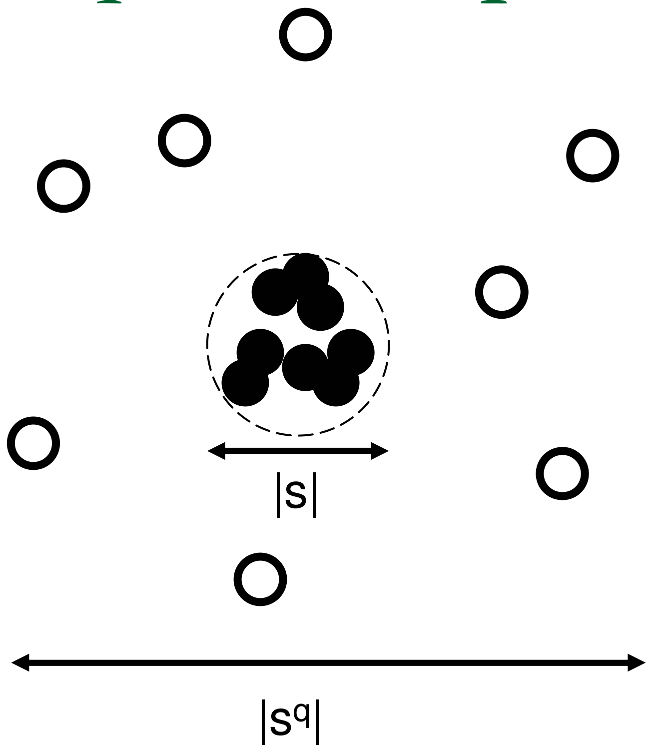
Many complex scenarios are possible:

- Small and continuous changes;
- Large, random and infrequent changes;
- Large and frequent changes.

Assumption:
Here we assume that changes are only slight in a dynamic environment. It would be beneficial to use knowledge about the old environment to help search in the new environment.

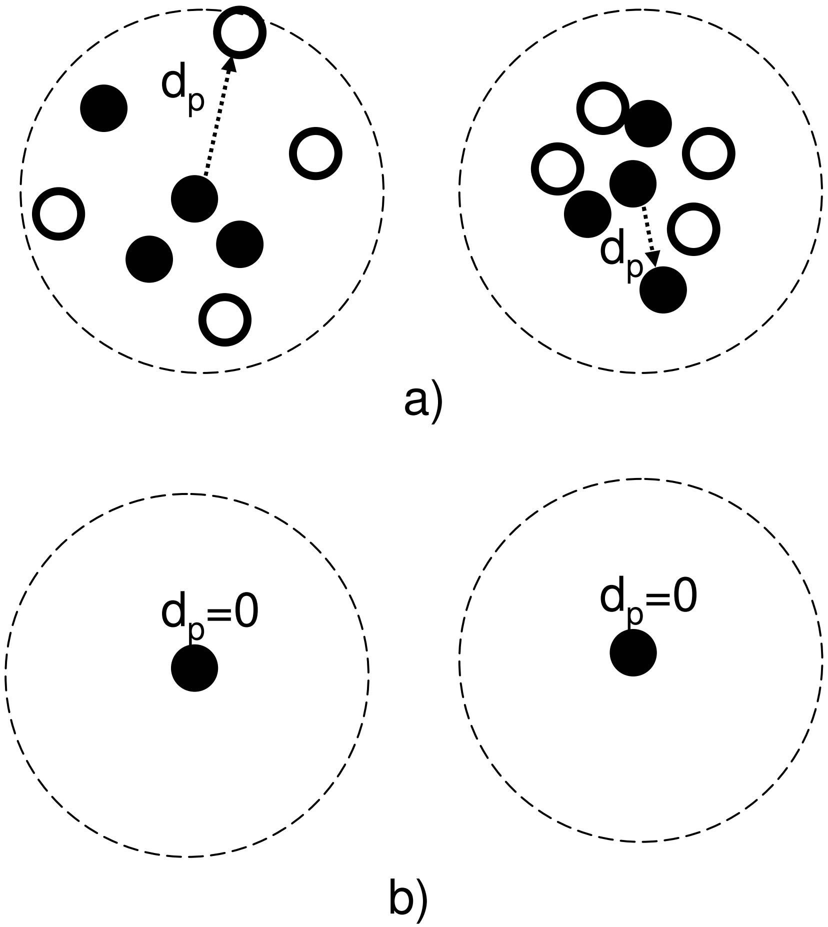
- Speciation-based PSO is able to identify peaks and converge onto these peaks in parallel and adaptively.
- It can be further enhanced by other techniques (eg., quantum swarms) to better track changing optima.

SPSO with quantum particles



In this quantum swarm model, a swarm is made up of neutral (ie., conventional and quantum particles. Quantum particles are positioned as a *cloud* centered around the $\vec{p}_g$, providing a constant level of particle diversity within a species (Li *et al.*, 2006).

SPSO with quantum particles



To see if a species has converged, we check if the particle diversity, d_p, of a species is smaller than a threshold.

To regain diversity, all particles except the species seed in the converged species are replaced by the same quantity of particles, centered around the species seed, with 50% as neutral particles and the remaining 50% as quantum particles.

Test functions for dynamic optimization

Juergen Branke's **Moving peak test functions** - The moving peak benchmark (MPB) is widely used in the EC community. A few recent PSO works also adopted it (Clerc, 2006; Blackwell and Branke, 2004; Li et al., 2006). For more information, refer to:

http://www.aifb.uni-karlsruhe.de/~jbr/MovPeaks/

Morrison and De Jong's **DF1** function generator – one of the early dynamic test function generator proposed (Morrison, 2005). A few authors have used it (Parrott and Li, 2006).

A few other dynamic test functions have also been proposed in recent years.

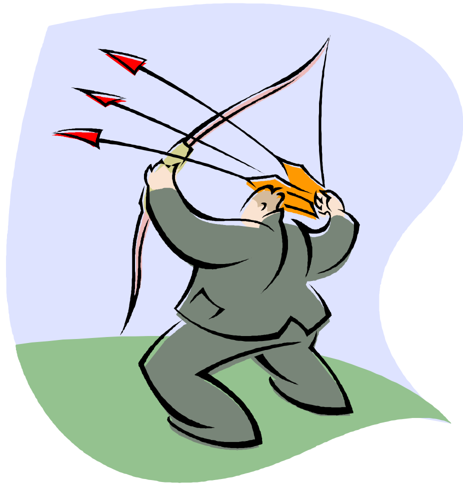
A demonstration run of SPSO tracking the global peak in a 10 peaks dynamic environment (Moving peaks Scienario2). Refer to (Li, et al. 2006) for details.

Multiobjective optimization

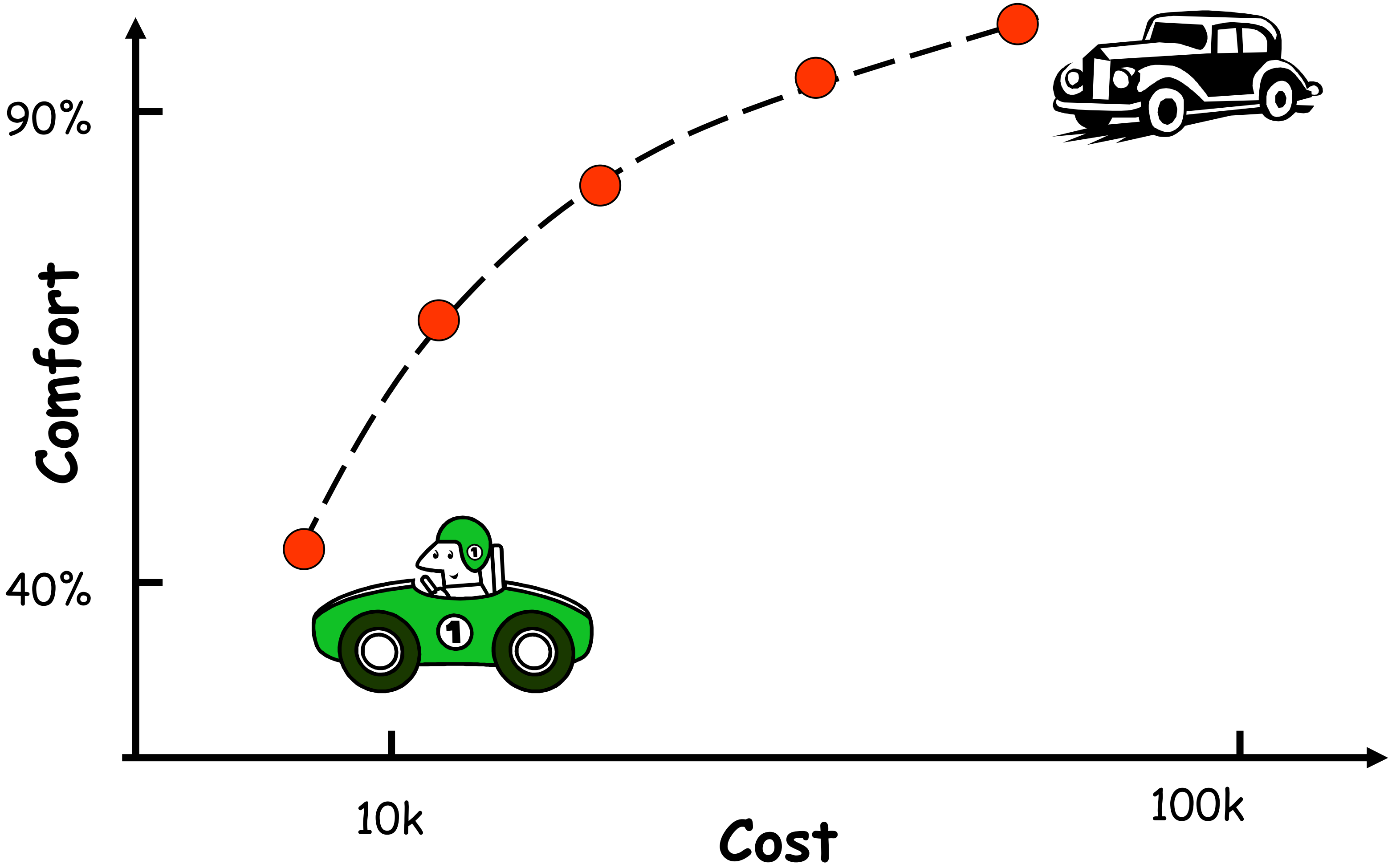
"The great decisions of human life have as a rule far more to do with the instincts and other mysterious unconscious factors than with conscious will and well-meaning reasonableness. The shoe that fits one person pinches another; there is no recipe for living that suits all cases. Each of us carries his own life-form - an indeterminable form which cannot be superseded by any other."

Carl Gustav Jung, Modern Man in Search of a Soul, 1933, p. 69

Many real-world problems involve multiple conflicting objectives, which need to be optimized simultaneously. The task is to find the best possible solutions which still satisfy all objectives and constraints. This type of problems is known as multiobjective optimization problems.



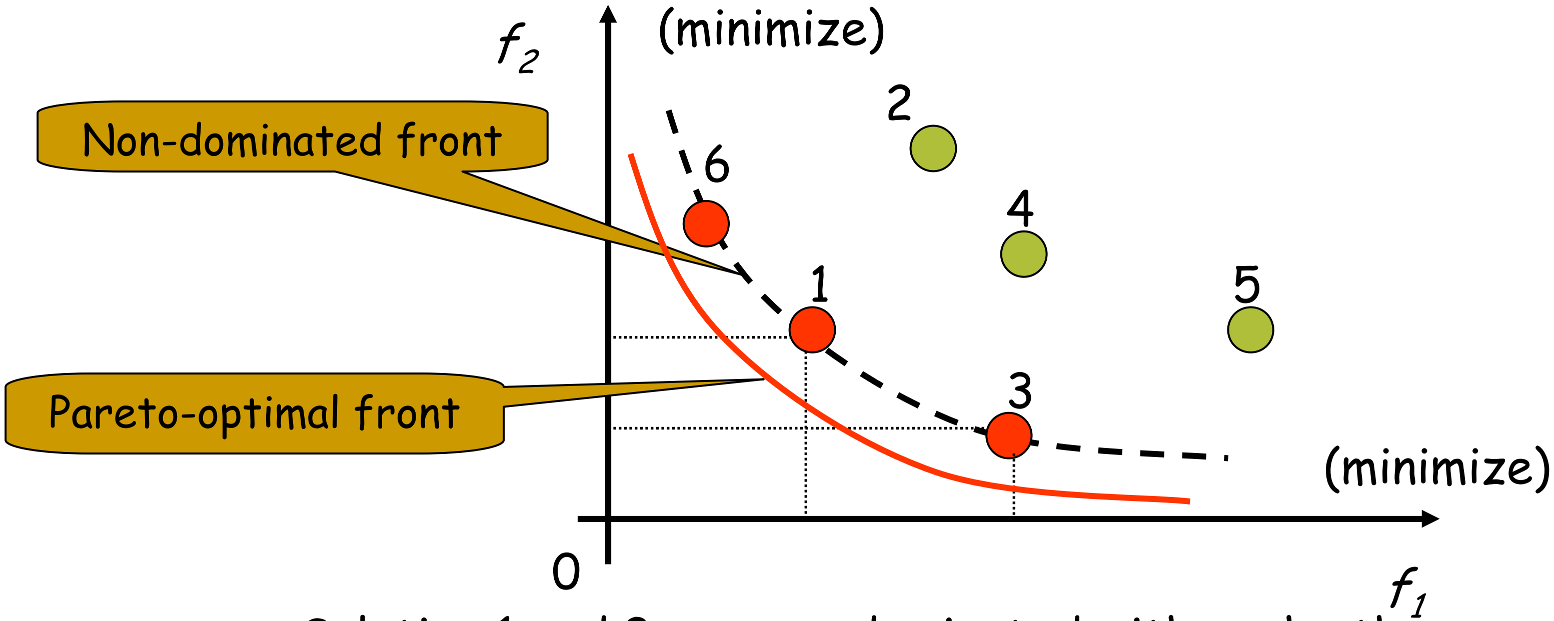
Multiobjective optimization



Concept of domination

A solution vector **x** is said to dominate the other solution vector **y** if the following 2 conditions are true:

- The solution **x** is no worse than **y** in all objectives;
- The solution **x** is strictly better than **y** in at least one objective.



Solution 1 and 3 are non-dominated with each other.

Solution 6 dominates 2, but not 4 or 5.

PSO for Multiobjective Optimization

Two major goals in multiobjective optimization:

- To obtain a set of non-dominated solutions as closely as possible to the true Pareto front;
- To main a well-distributed solution set along the Pareto front.

Some earlier PSO models using different techniques:

MOPSO (Coello et al., 2002) – dominance comparison for each particle with its personal best; diversity is maintained using a grid-based approach.

Aggregation approaches (Parsopoulos and Vrahatis, 2002) – 3 different aggregation functions used.

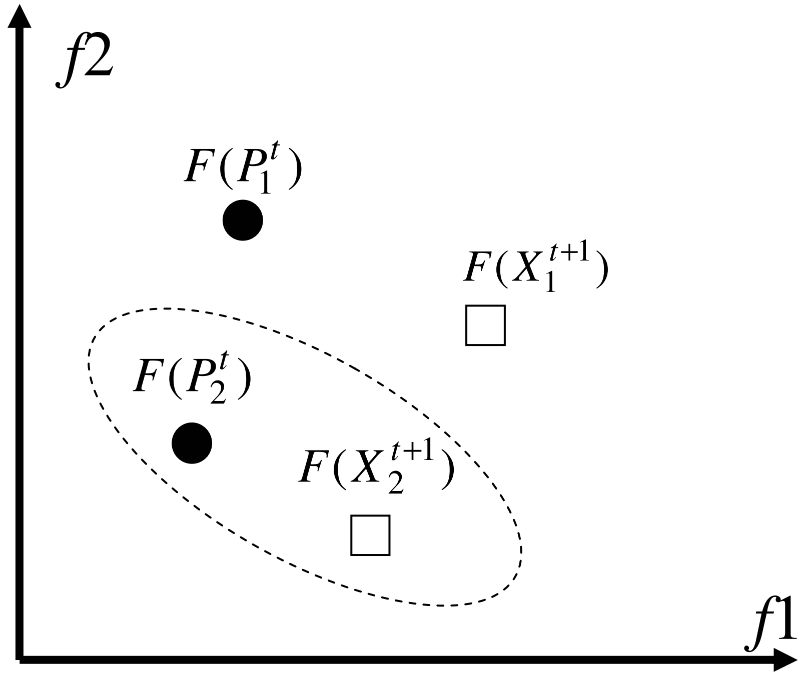
Fieldsend and Sigh (2002) – use “dominated tree” to store non-dominated solutions.

Dynamic neighbourhood (Hu and Eberhart, 2002, 2003) – One objective optimized at a time, later enhanced with an “extended memory”.

Sigma method (Mostaghim & Teich, 2003) – a method to better choose local guides

Non-dominated Sorting PSO (Li, 2003) – dominance comparison for all particles including personal bests; non-dominated sorting is used, similar to NSGA II.

Better dominance comparison for PSO



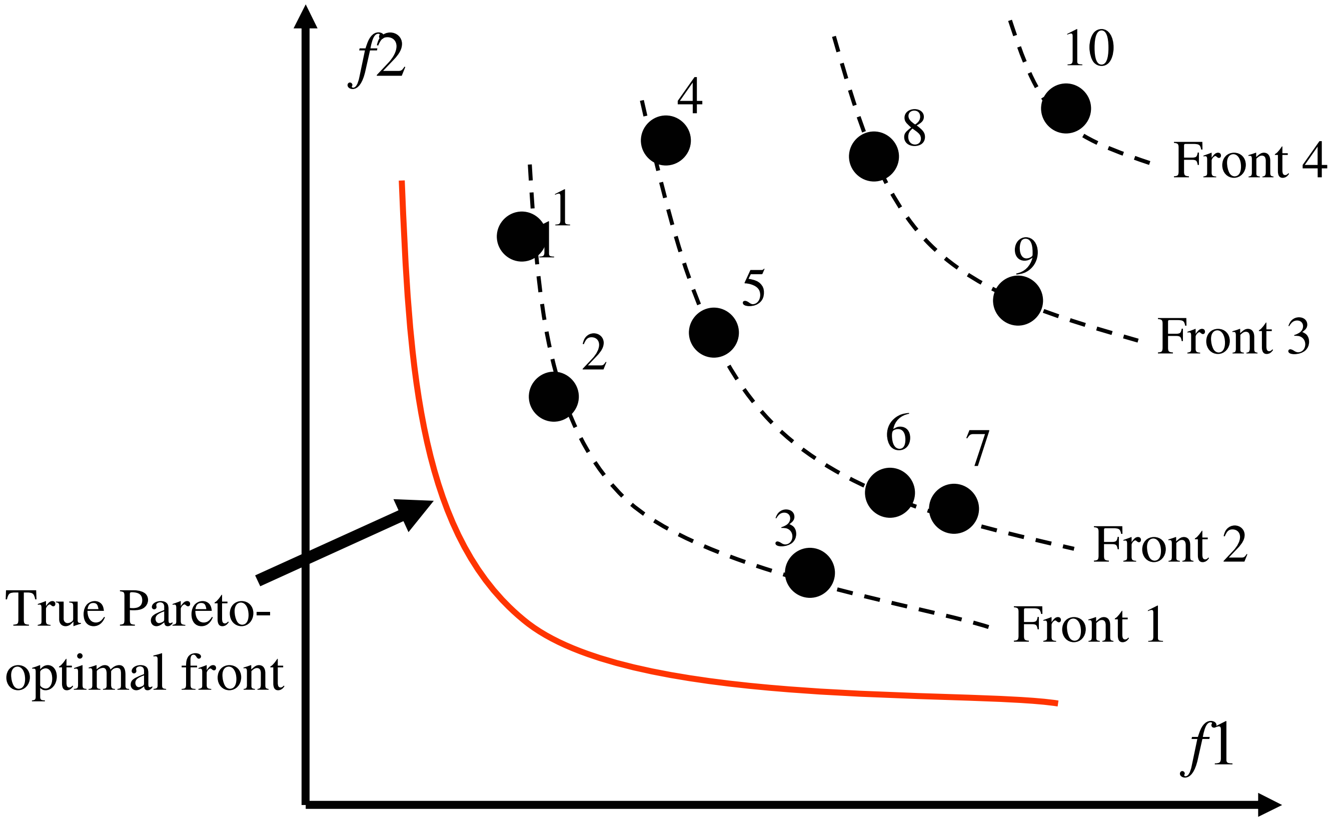
Dominance relationships among 4 particles, including the personal bests of two particles, and their potential offspring, assuming minimization of *f1* and *f2*.

NSPSO Algorithm

The basic idea:

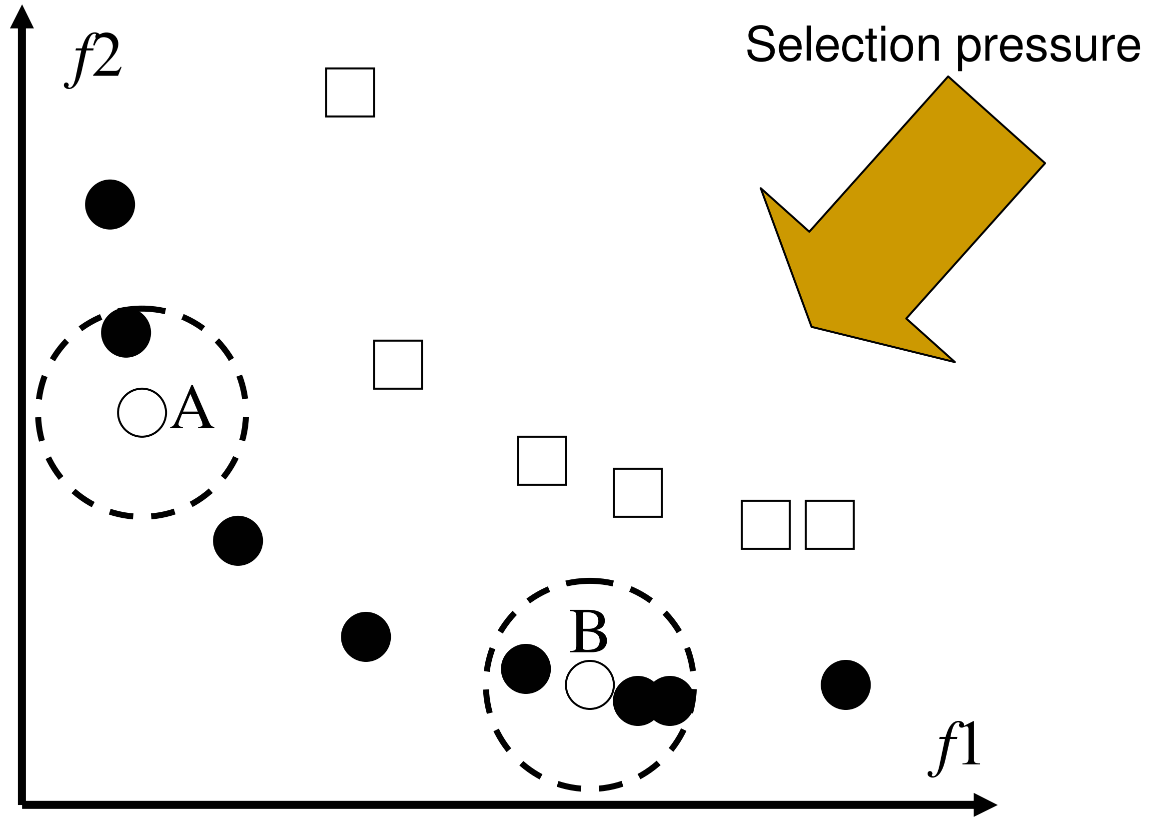
- Instead of comparing solely on a particle's personal best with its potential offspring, the entire population of **N** particles' personal bests and **N** of these particles' offspring are first combined to form a temporary population of 2**N** particles. After this, domination comparisons among all the 2**N** individuals in this temporary population are carried out.
- Sort the entire population in different non-domination levels (as in NSGA II). This type of sorting can then be used to introduce the selection bias to the individuals in the populations, in favour of individuals closer to the true Pareto front.
- At each iteration step, we choose only **N** individuals out of the 2**N** to the next iteration step, based on the non-domination levels, and two niching methods.

Non-dominated Sorting PSO



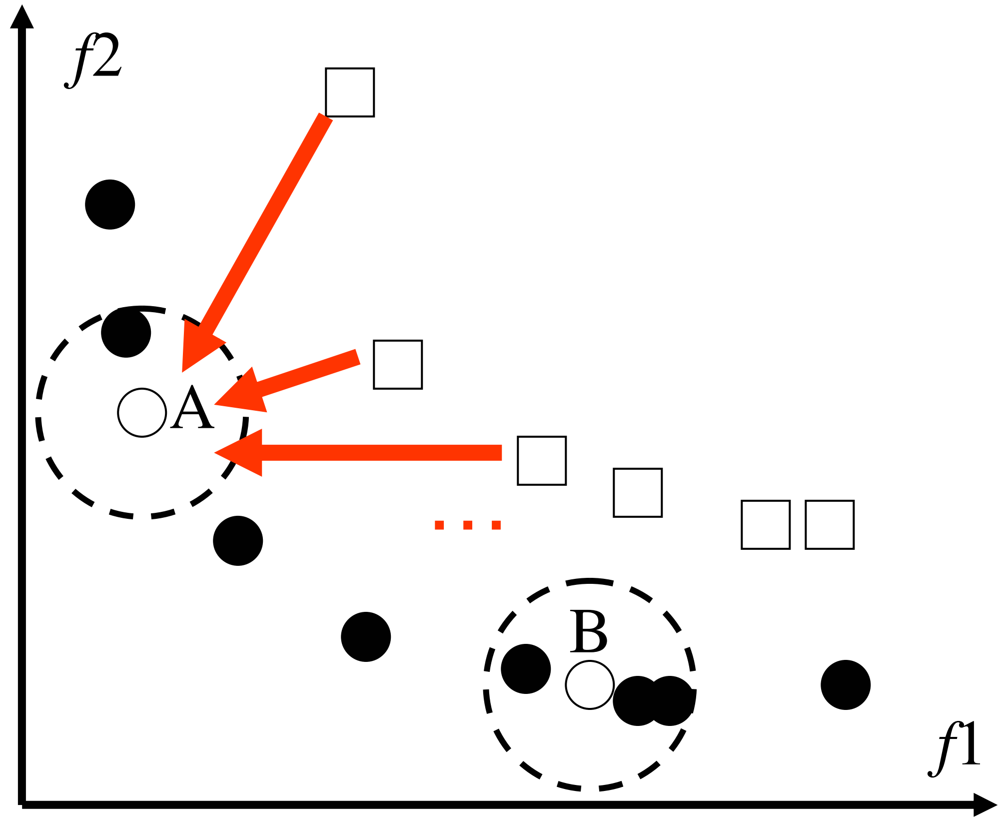
Selection pressure towards the true Pareto-optimal front.

Niching techniques



A will be preferred over B, since A has a smaller niche count than B.

Selecting better guides



Particles in the "less-crowded' area of the non-dominated front is more likely to be chosen as $\vec{p}_g$ for particles in the population, eg., **A** is more likely than **B**.

Performance metrics

- Diversity of the solutions along the Pareto front in the final population:

$$\Delta = \frac{\sum_{m=1}^{M} d_m^e + \sum_{i=1}^{|Q|} | d_i - \bar{d} |}{\sum_{m=1}^{M} d_m^e + |Q| \bar{d}},$$

- Number of non-dominated solutions found;
- Closeness to the true Pareto-optimal front:

$$GD = \frac{(\sum_{i=1}^{|Q|} d_i^p)^{1/p}}{|Q|}.$$

Test functions (ZDT series)

Two objectives are to be minimized:

$$\begin{aligned} &Minimize \quad f_1(x) \\ &Minimize \quad f_2(x) = g(x)h(f_1(x), g(x)). \end{aligned}$$

In all problems except ZDT5, the Pareto-optimal front is formed with g(x) = 1

Note that more scalable test functions, such as the DTLZ functions (with more than 2 objectives) were also proposed.

ZDT series

ZDT1

$$f_1(x) = x_1,$$

$$g(x) = 1 + 9\left(\sum_{i=2}^{n} x_i\right)/(n-1)$$

$$h(f_1, g) = 1 - \sqrt{f_1 / g}.$$

ZDT3

$$f_1(x) = x_1,$$

$$g(x) = 1 + 9\left(\sum_{i=2}^{n} x_i\right)/(n-1)$$

$$h(f_1, g) = 1 - \sqrt{f_1 / g} - (f_1 / g)\sin(10\pi f_1).$$

ZDT2

$$f_1(x) = x_1,$$

$$g(x) = 1 + 9\left(\sum_{i=2}^{n} x_i\right)/(n-1)$$

$$h(f_1, g) = 1 - (f_1 / g)^2.$$

ZDT4

$$f_1(x) = x_1,$$

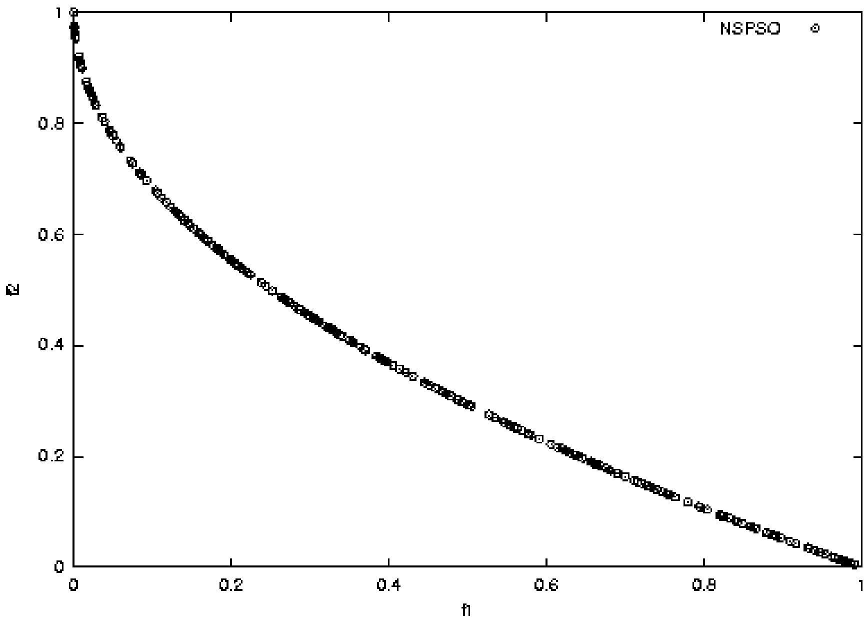
$$g(x) = 1 + 10(n-1) + \left(\sum_{i=2}^{n} (x_i^2 - 10\cos(4\pi x_i)\right)),$$

$$h(f_1, g) = 1 - \sqrt{f_1 / g}.$$

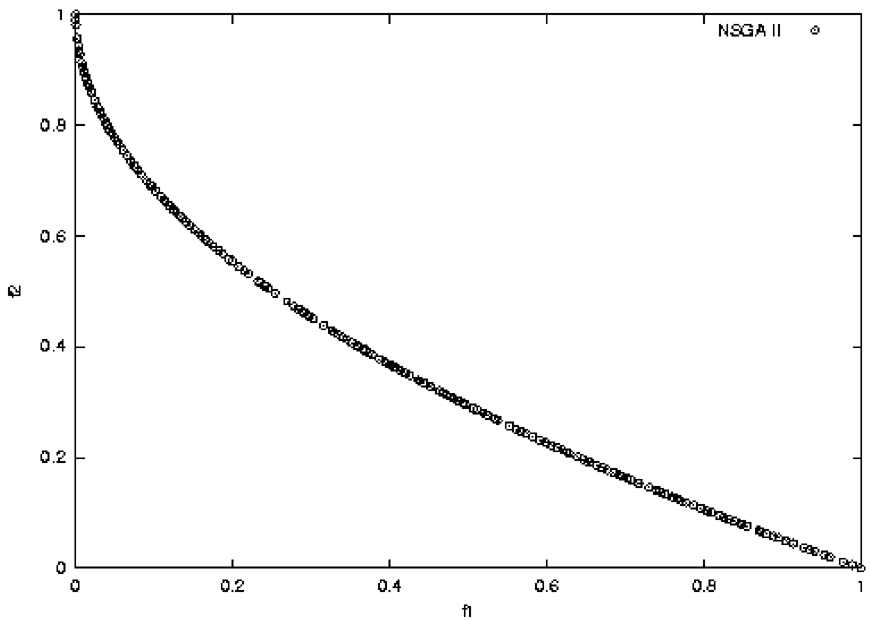
Note: n= 30 (30 variables); x_i in the range [0,1], except for ZDT4, where x_2- x_{30} lie in the range [-5, 5].

Experimental results

NSPSO



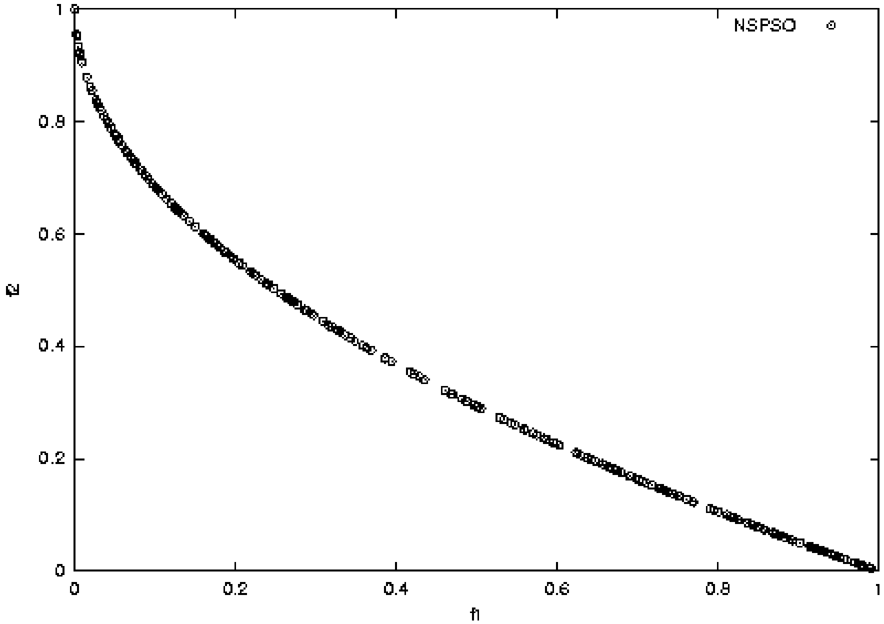
NSGA II



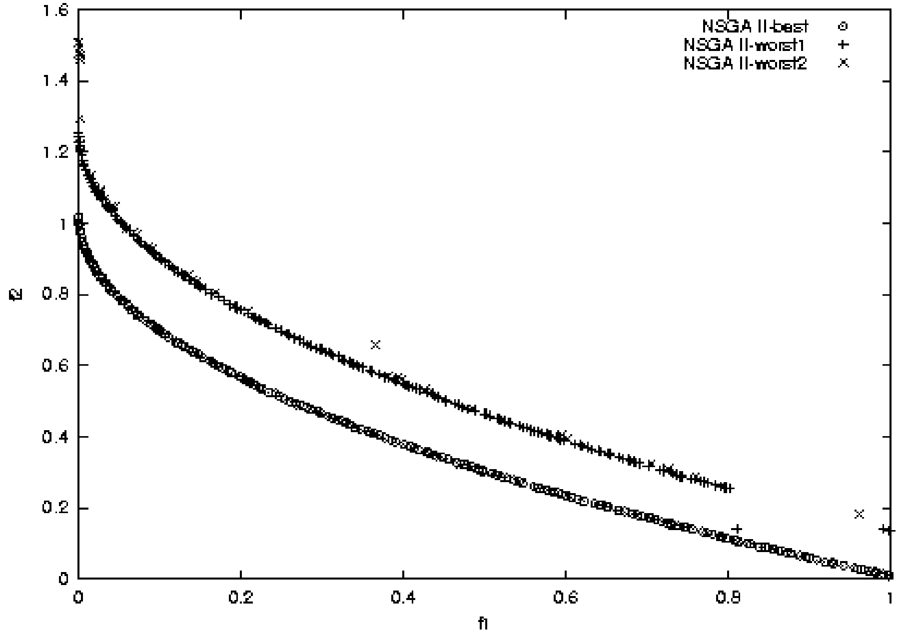
Non-dominated solutions found for ZDT1.

Experimental results

NSPSO

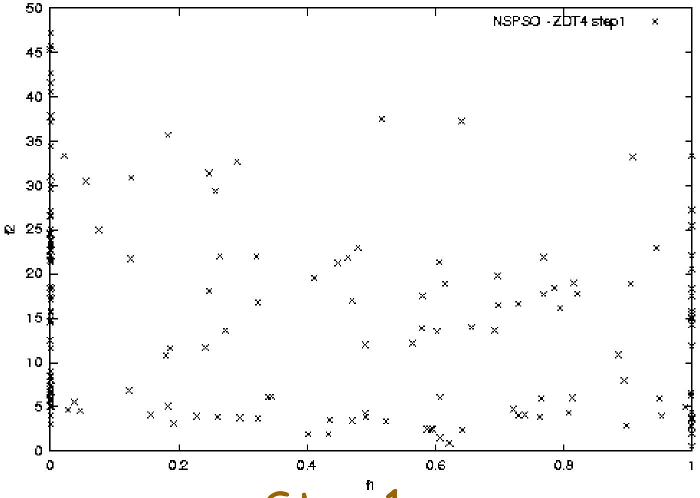


NSGA II

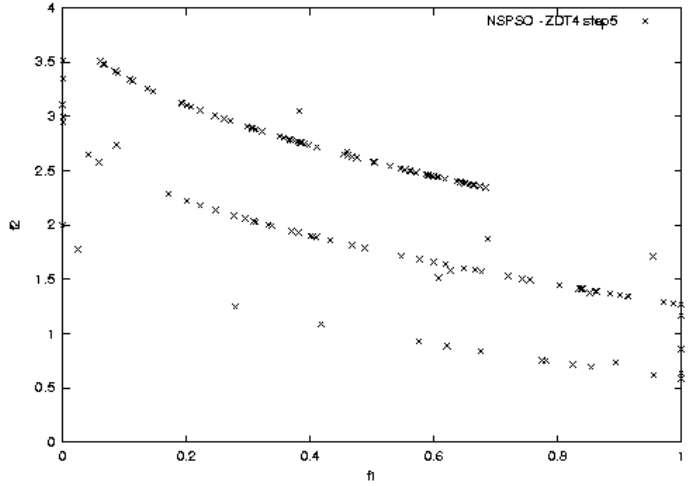


Non-dominated solutions found for ZDT4.

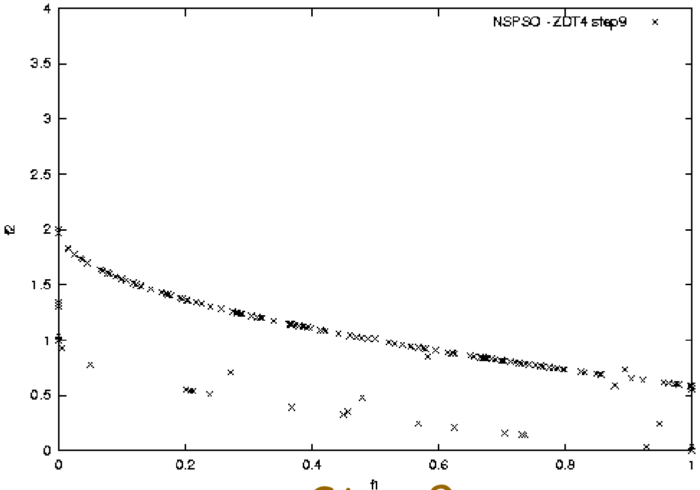
Snapshots of a NSPSO run on ZDT4



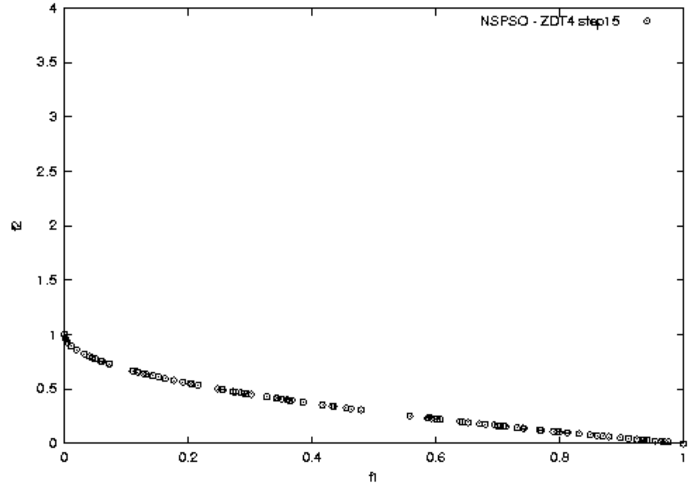
Step 1



Step 3



Step 9



Step 15

Constraint handling

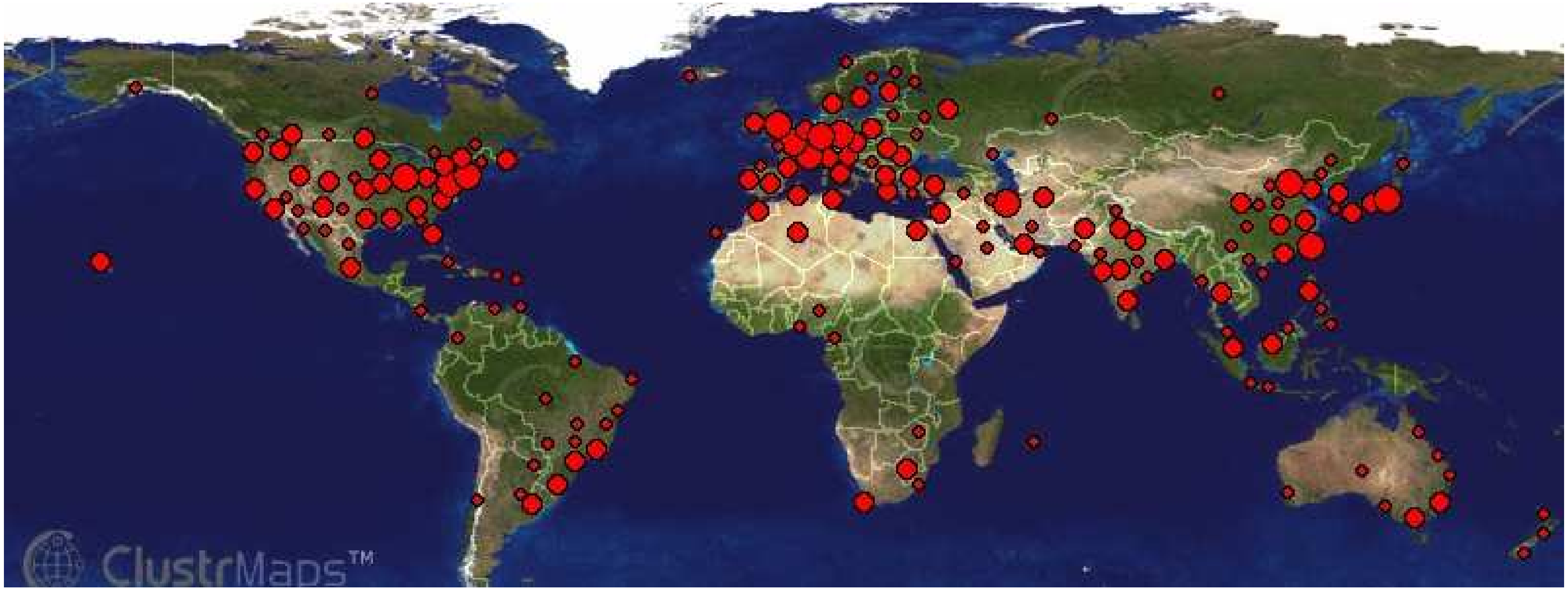
The most common approach for solving constrained problems is the use of a penalty function. The constrained problem is transformed into an unconstrained one, by penalizing the constraints and creating a single objective function.

- Non-stationary penalty functions (Parsopoulos and Vrahatis, 2002):
  - A penalty function is used, and the penalty value is dynamically modified during a run. This method is problem dependent, however, its results are generally superior to those obtained through stationary functions.
- Preservation of feasible solutions (Hu and Eberhart, 2002):
  - During initialization, all particles are repeatedly initialized until they satisfy all constraints; when calculating personal best and global best, only those positions in feasible space are counted.
- Based on closeness to the feasible region (Toscano and Coello, 2004):
  - If both particles compared are infeasible, then the particle that has the lowest value in its total violation of constraints wins.

Please see A/Prof. Ponnuthurai Suganthan's tutorial for further information on PSO for constraint handling.

More information

Particle Swarm Central: http://www.particleswarm.info



Visitors' hits since 12 June 2006 (updated daily).

References (incomplete)

Background:

1) Reynolds, C.W.: Flocks, herds and schools: a distributed behavioral model. *Computer Graphics*, 21(4), p.25-34, 1987.
2) Heppner, F. and Grenander, U.: A stochastic nonlinear model for coordinated bird flocks. In S. krasner, Ed., The Ubiquity of Chaos. AAAS Publications, Washington, DC, 1990.
3) Kennedy, J. and Eberhart, R.: Particle Swarm Optimization. In *Proceedings of the Fourth IEEE International Conference on Neural Networks*, Perth, Australia. IEEE Service Center(1995) 1942-1948.
4) Kennedy, J., Eberhart, R. C., and Shi, Y., *Swarm intelligence*, San Francisco: Morgan Kaufmann Publishers, 2001.
5) Clerc, M.: *Particle Swarm Optimization*, ISTE Ltd, 2006.

References – continued…

New improvements and variants:

1) Y. Shi and R. C. Eberhart, “A modified particle swarm optimizer,” in Proc. IEEE Congr. Evol. Comput., 1998, pp. 69–73.
2) Clerc, M. and Kennedy, J.: The particle swarm-explosion, stability and convergence in a multidimensional complex space. *IEEE Trans. Evol. Comput.* Vol.**6**, no.2, pp.58-73, Feb. 2002.
3) Kennedy, J., and Mendes, R. (2002). Population structure and particle swarm performance. Proc. of the 2002 World Congress on Computational Intelligence.
4) T. Krink, J. S. Vesterstroem, and J. Riget, “Particle swarm optimization with spatial particle extension,” in Proc. Congr. Evolut. Comput., Honolulu, HI, 2002, pp. 1474–1479.
5) M. Lovbjerg and T. Krink, “Extending particle swarm optimizers with self-organized criticality,” in Proc. Congr. Evol. Comput., Honolulu, HI, 2002, pp. 1588–1593.
6) X. Xie, W. Zhang, and Z. Yang, “A dissipative particle swarm optimization,” in Proc. Congr. Evol. Comput., Honolulu, HI, 2002, pp.1456–1461.
7) T. Peram, K. Veeramachaneni, and C. K. Mohan, “Fitness-distance-ratio based particle swarm optimization,” in Proc. Swarm Intelligence Symp., 2003, pp. 174–181.
8) Kennedy, J.: Bare bones particle swarms. In Proc. of the Swarm Intelligence Symposium (SIS 2003), 2003.
9) Mendes, R. (2004). Population Topologies and Their Influence in Particle Swarm Performance. PhD Thesis, Universidade do Minho, Portugal.
10) R. Mendes, J. Kennedy, and J. Neves, “The fully informed particle swarm: Simpler, maybe better,” *IEEE Trans. Evol. Comput*., vol. 8, pp.204–210, Jun. 2004.
11) F. van den Bergh and A.P. Engelbrecht: A cooperative approach to particle swarm optimization. *IEEE Trans. Evol. Comput*., vol.8, pp.225-239, Jun. 2004.
12) A. Ratnaweera, S. Halgamuge, and H. Watson, “Self-organizing hierarchical particle swarm optimizer with time varying accelerating coefficients,” *IEEE Trans. Evol. Comput*., vol. 8, pp. 240–255, Jun. 2004.
13) J.J. Liang, A.K.Qin, P.N. Suganthan, and S. Baskar: Comprehensive Learning Particle Swarm Optimizer for Global Optimization of Multimodal Functions. *IEEE Trans. Evol. Comput.*, vol.10, No.3, pp.281 – 295, Jun. 2006.

References – continued…

Speciation and niching:

1) A. Petrowski, "A clearing procedure as a niching method for Genetic Algorithms," in Proc. of the 1996 IEEE International Conference on Evolutionary Computation, 1996, pp.798–803.
2) R. Brits, A.P. Engelbrecht, and F. van den Bergh, "A niching particle swarm optimizer," in Proc. of the 4th Asia-Pacific Conference on Simulated Evolution and Learning 2002 (SEAL 2002), 2002, pp.692–696.
3) J.P. Li, M.E. Balazs, G. Parks and P.J. Clarkson, "A species conserving genetic algorithm for multimodal function optimization," Evol. Comput., vol.10, no.3, pp.207–234, 2002.
4) X. Li, "Adaptively choosing neighbourhood bests using species in a particle swarm optimizer for multimodal function optimization," in Proc. of Genetic and Evolutionary Computation Conference 2004 (GECCO'04), LNCS 3102, eds. Deb, K. et al., 2004, pp.105–116.
5) K.E. Parsopoulos and M.N. Vrahatis, "On the computation of all global minimizers through Particle Swarm Optimization," IEEE Trans. Evol. Comput., vol.8, no.3, Jun. 2004, pp.211–224.
6) Bird, S. and Li, X.(2006), "Adaptively Choosing Niching Parameters in a PSO", in Proceeding of Genetic and Evolutionary Computation Conference 2006 (GECCO'06), eds. M. Keijzer, et al., p.3 - 9, ACM Press.
7) Bird, S. and Li, X.(2006), "Enhancing the robustness of a speciation-based PSO", in Proceeding of Congress of 2006 Evolutionary Computation (CEC'06), p.3185 - 3192, IEEE Service Center, Piscataway, NJ 08855-1331.

References – continued…

Optimization in dynamic environments:

1) R. C. Eberhart and Y. Shi. Tracking and optimizing dynamic systems with particle swarms. In Proc. the 2001 Congress on Evolutionary Computation CEC2001, p.94–100. IEEE Press, 27-30 May 2001.
2) J. Branke, Evolutionary Optimization in Dynamic Environments. Norwell, MA: Kluwer Academic Publishers, 2002.
3) A. Carlisle and G. Dozier. Tracking changing extrema with adaptive particle swarm optimizer. In Proc. World Automation Cong.,, pages 265–270, Orlando FL USA, 2002.
4) X. Hu and R. Eberhart. Adaptive particle swarm optimisation: detection and response to dynamic systems. In Proc. Congress on Evolutionary Computation, p.1666–1670, 2002.
5) T. Blackwell and P. Bentley. Dynamic search with charged swarms. In Proc. the Workshop on Evolutionary Algorithms Dynamic Optimization Problems (EvoDOP-2003), pages 19–26, 2002.
6) T. Blackwell and J. Branke. Multi-swarm optimization in dynamic environments. In LNCS, No. 3005, Proc. Of Applications of Evolutionary Computing: EvoWorkshops 2004: EvoBIO, EvoCOMNET, EvoHOT, EvoISAP, EvoMUSART, and EvoSTOC, pages 489–500, 2004.
7) D. Parrott and X. Li, “A particle swarm model for tacking multiple peaks in a dynamic environment using speciation,” in Proc. of the 2004 Congress on Evolutionary Computation, 2004, pp.98–103.
8) T. Blackwell and J. Branke. Multi-swarms, exclusion, and anti-convergence in dynamic environments. *IEEE Trans. on Evol. Compu.*, Vol.**10**, No.4, August 2006, pp.459-472.
9) Parrott, D. and Li, X. (2006), "Locating and Tracking Multiple Dynamic Optima by a Particle Swarm Model using Speciation", *IEEE Trans on Evol. Compu.*, Vol.**10**, No.4, August 2006, pp.440-458.
10) Li, X., Branke, J. and Blackwell, T. (2006), "Particle Swarm with Speciation and Adaptation in a Dynamic Environment ", in Proceeding of Genetic and Evolutionary Computation Conference 2006 (GECCO'06), eds. M. Keijzer, et al., p.51 - 58, ACM Press.

References – continued…

Multiobjective optimization:

1) Deb, K.: Multi-Objective Optimization using Evolutionary Algorithms, John Wiley & Sons, Chichester, UK (2001).
2) Deb, K., Agrawal,S., Pratap, A. and Meyarivan, T.: A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE Transactions on Evolutionary Computation 6(2): 182-197 (2002).
3) Hu, X. and Eberhart, R.: Multiobjective Optimization Using Dynamic Neighbourhood Particle Swarm Optimization. In Proceedings of the IEEE World Congress on Computational Intelligence, Hawaii, May 12-17, 2002. IEEE Press (2002).
4) Coello, C.A.C. and Lechuga, M.S.: MOPSO: A Proposal for Multiple Objective Particle Swarm Optimization, in Proceedings of Congress on Evolutionary Computation (CEC'2002), Vol. 2, IEEE Press (2002) 1051-1056.
5) Mostaghim, S. and Teich, J.: Strategies for finding good local guides in Multi-Objective Particle Swarm Optimization (MOPSO. In Proc. 2003 IEEE Swarm Intelligence Symp., Indianapolis, IN, Apr. 2003, pp.26-33.
6) Fieldsend, J.E. and Singh, S.: A multi-objective algorithm based upon particle swarm optimization, an efficient data structure and turbulence. In Proc. 2002 U.K. Workshop on Computational Intelligence, Birmingham, U.K., Sept. 2002, pp.37-44.
7) Li, X.: A Non-dominated Sorting Particle Swarm Optimizer for Multiobjective Optimization, in Erick Cant-Paz et al. (editors), Genetic and Evolutionary Computation - GECCO 2003. Proceedings, Part I, Springer, LNCS Vol. 2723, (2003) 37-48.
8) C. A. C. Coello, G. T. Pulido, and M. S. Lechuga MS, “Handling multiple objectives with particle swarm optimization,” *IEEE Trans. Evol. Comput.*, vol. 8, no. 3, pp. 256–279, Jun. 2004.

References – continued…

Constraint handling:

1) Z. Michalewicz and M. Schoenauer. Evolutionary Algorithms for Constrained Parameter Optimization Problems. *Evolutionary Computation*, 4(1):1–32, 1996.
2) T. P. Runarsson and X. Yao. Stochastic Ranking for Constrained Evolutionary Optimization. *IEEE Transactions on Evolutionary Computation*, 4(3):284–294, September 2000.
3) X. Hu, and R. Eberhart. Solving constrained nonlinear optimization problems with particle swarm optimization. 6th World Multiconference on Systemics, Cybernetics and Informatics (SCI 2002), Orlando, USA.
4) K. Parsopoulos and M. Vrahatis. Particle Swarm Optimization Method for Constrained Optimization Problems. In P. Sincak, J.Vascak, V. Kvasnicka, and J. Pospicha, editors, Intelligent Technologies - Theory and Applications: New Trends in Intelligent Technologies, pages 214–220. IOS Press, 2002. Frontiers in Artificial Intelligence and Applications series, Vol. 76 ISBN: 1-58603-256-9.
5) G. Coath and S. K. Halgamuge. A comparison of constraint-handling methods for the application of particle swarm optimization to constrained nonlinear optimization problems. In Proceedings of the 2003 Congress on Evolutionary Computation, p.2419 - 2425. IEEE, December 2003.
6) J. Zhang and F. Xie. DEPSO: Hybrid particle swarm with differential evolution operator. In Proceedings of IEEE International Conference on Systems, Man and Cybernetics, p.3816-3821. IEEE, October 2003.
7) G. Toscano and C. Coello. A constraint-handling mechanism for particle swarm optimization. In Proceedings of the 2004 Congress on Evolutionary Computation, p.1396 - 1403. IEEE, June 2004.